The University Tutorial Series

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THE NEW

MATRICULATION SOUND

The University Tutorial Series

THE NEW MATRICULATION SOUND

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PREFACE.

EVEN in the most elementary book on *Sound* it is essential that some exposition of the difficult subjects, *vibratory motion* and *wave motion*, should be attempted. In most elementary books it has been usual to give this exposition in a few incidental articles introduced as digressions from the main line of treatment. The author, however, is of opinion that the difficulty and importance of these fundamental subjects not only justify but demand their systematic exposition as a preliminary to the study of *Sound.*

In this book, therefore, the first six chapters are devoted to a simple exposition of vibratory motion and wave motion. The treatment throughout is of a strictly elementary character, but it necessarily involves difficulties inherent in the nature of the subjects. An effort, however, has been made to minimise these difficulties, and to present them to the reader in an easy and simple form suitable to the scope of the book.

From Chapter VII. onwards the book deals with the elements of *Sound* proper. The author has attempted to give a simple experimental presentment of the first principles of the subject, and as vibratory motion and wave

motion have already been dealt with, it has been possible to do this without interrupting the continuity of the treatment by distracting digressions. Throughout the book the more difficult parts are printed in small type. These parts may be omitted on first reading.

The book is written primarily for the use of students preparing for the Matriculation Examination of London University, but it is hoped that it may be found useful as a general introduction to the study of *Sound.* Candidates for London University Matriculation who wish to follow the minimum course of reading may omit §§ 10, 11, 42, 54, 55, 62, 75, 77-79, 82, 92-95, 104, Chapter XVI., and all small type, with the exception of experiments, throughout the book.

R. W. S.

CONTENTS.

SOUND.

CHAPTER I.

SIMPLE HARMONIC MOTION.

1. Uniform motion in a circle. When a body moves with uniform speed *in a straight line* it is not subject to acceleration, for the velocity remains constant both in *magnitude* and in *direction*. When, however, a body moves with uniform speed, but with change of direction, as when it moves *in a circle*, it is subject to acceleration, for although the magnitude of the velocity is constant, the direction is subject to continuous change.

The acceleration when a velocity, constant in magnitude, suffers a *small* change in direction is easily determined.

Let a velocity of v units, represented by O A (Fig. 1) in magnitude and direction, change direction through a *small* angle a in a time t so that at the end of the time it is represented by O B in magnitude and direction. Then, by the triangle of velocities, A B represents the *change of velocity* in the time t, and as A B represents $v\,a$ units of velocity, then $v\,a/t$ measures the average acceleration for the time t. When a and t are very small $v\,a/t$ gives the acceleration at a particular instant, and the direction of this acceleration, indicated in the figure by A B, is evidently at right angles to the direction of the velocity at that instant.

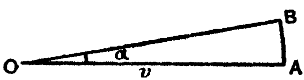


Fig. 1.

Now when a body moves with uniform speed in a circle of radius r (Fig. 2) the direction of the velocity changes through an angle a in the time that the body takes to move over a distance ra on the circle. If the velocity of the body is v units, this time is evidently $r\,a/v$, and substituting this for t in the result obtained above, the acceleration is found to be $(va)/(ra/v)$ or v^2/r. Since v and r are both constant the magnitude of this acceleration is also constant, and its direction, being always at right angles to the direction of the velocity, is always towards the centre of the circle. That is, the acceleration of a body moving with a uniform speed v in a circle of radius r is of constant magnitude v^2/r and is directed towards the centre of the circle. This, of course, means that in order to make a body move with uniform speed in a circle it must be acted on by a constant force directed towards the centre of the circle.

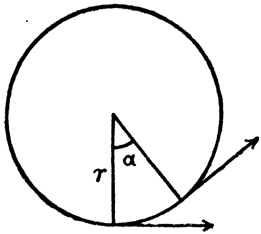


Fig. 2.

2. Simple harmonic motion. In the circle A P B (Fig. 3) take any point P. Draw *any* diameter, such as A B, and drop a perpendicular, P p, from the point P on to this diameter. Then the point p, the foot of this perpendicular, is the *projection* of the point P on the diameter A B. Now imagine the point P to move with uniform speed round the circle and consider the motion of p, the projection of this point on the diameter A B, along that diameter. If the motion take place in the direction of the arrow, then it is evident from the figure that as P passes round through C, A, D, and B back to P, the point p moves forward along B A through O up to A and then back through O to B and then forward to its initial position.

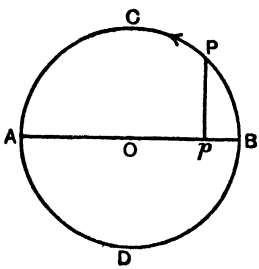


Fig. 3.

Similarly, it will be seen that if P starts from any point on the circle and describes a complete revolution, p moves

to and fro along A B, from its starting point up to A, then to B, and then back to its starting point.

That is, if the point P moves continuously round the circle with uniform speed, then the point p oscillates backwards and forwards between A and B along the diameter AB in a motion which is known as *simple harmonic motion*. Hence, in general terms, if a point move round a circle with uniform speed, the projection of that point on any diameter of the circle moves in *simple harmonic motion* along that diameter.

Since the point P moves with uniform speed round the circle it describes each complete revolution in a constant period of time. Hence it follows that the point p, moving in simple harmonic motion, describes each complete to-and-fro movement in the same constant period of time. This period of time is known briefly as the *period* of the motion. That is, the time in which a point moving in simple harmonic motion describes a complete to-and-fro movement or vibration is the *period* of its motion.

If r denote the radius of the circle and v the velocity of P, then $2\pi r/v$ is evidently the time of a complete revolution and also the *period* of the simple harmonic motion of p.

If the velocity of P is specified as angular velocity, ω, round O, then in the usual way $v = r\omega$ and the *period* of the motion is given by $2\pi r/r\omega$ or $2\pi/\omega$.

If the motion of a point moving in simple harmonic motion along a line A B be considered relative to O, the centre of its path of motion, it will be seen that each complete to-and-fro movement of the point consists of motion from O to A, then from A to B, and then back to O, and that the continuous motion consists merely of a sequence of these complete to-and-fro movements, each complete movement occupying a constant period of time. That is, simple harmonic motion is of a *periodic* character.

If O be considered as the position of rest of the point, then, when in simple harmonic motion along A B, the greatest distance it moves from its position of rest is represented by O A or O B. This greatest distance is known as the *amplitude* of the motion. That is, in the case

of a point moving along a line in simple harmonic motion the greatest distance it moves from the centre of its path is the *amplitude* of its motion.

The distance of a point, p, in simple harmonic motion, from the centre of its path of motion, O, as represented by O p, is known as the *displacement* of the point, and is evidently equal at any instant to O P cos P O B or O P sin P O C. If P moves with angular velocity ω, then at any time t after the point p leaves O the angle P O C $= \omega t$ and the displacement of p is $r \sin \omega t$, where r denotes the amplitude of the motion.

3. Velocity and acceleration of a point moving in simple harmonic motion. When a body moves with simple harmonic motion the velocity and acceleration of the point vary from point to point in the path in a manner characteristic of the motion.

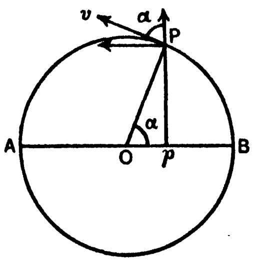


Fig. 4.

When the point p in Fig. 4 moves in simple harmonic motion along the line A O B its velocity at any point in its path is evidently the *component* parallel to A B of the velocity of P resolved in directions parallel to, and at right-angles to A B. Hence, when the position of p is such that the angle B O P (Fig. 4) is denoted by a, the velocity of p is given by $v \sin a$ and is therefore *proportional* to the length of the perpendicular Pp, which is given by $r \sin a$ where r is the radius of the circle. This indicates that as p moves to and fro along A B in simple harmonic motion its velocity increases and decreases in the same way as the length of the perpendicular Pp increases and decreases. Its velocity is therefore zero at the points A and B, and has a maximum value, v, equal to that of P at the point O.

Similarly, the acceleration of p at any point in its path is given by the component parallel to A B of the acceleration of P resolved parallel to, and at right angles to A B. The acceleration of P at any point in its path has been shown to be v^2/r towards the centre of the circle. Hence, when the position of p is such that the angle B O P (Fig. 5) is denoted by a the acceleration of p *towards the centre*, O, is given by $v^2/r \,.\, \cos a$, and is therefore *proportional* to the distance Op, which is equal to $r \cos a$. The acceleration of p is therefore always towards the centre O, and is directly proportional to the distance of

p from O. That is, the acceleration is zero when p is at O and increases, as the distance Op increases, to a maximum value v^2/r when p is at A or B.

If the velocity of P be specified as angular velocity, ω, round O, then the acceleration of p is evidently given by $\omega^2 r \cos\alpha$. But as $Op = r\cos\alpha$ this expression for the acceleration reduces to $\omega^2 . Op$ where Op is the distance of p from O. That is, if x denote the distance of the point from O the acceleration is always towards O and equal to $\omega^2 x$.

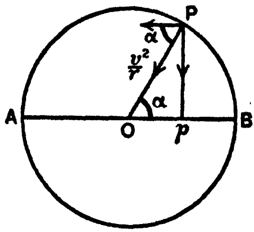


Fig. 5.

Further, in Art. 2 it has been shown that the period of the motion is given by $2\pi/\omega$. If this period be denoted by t we have $t = 2\pi/\omega$ or $\omega = 2\pi/t$, and therefore $\omega^2 x$, the acceleration at any point, may be written as $4\pi^2 x/t^2$. That is, if a denote the acceleration of a point moving in simple harmonic motion when at a distance x from its centre of motion, then $a = 4\pi^2 x/t^2$ where t is the period of the motion. This gives a relation from which t can be determined when the value of a for a given value of x is known, for we get $t = 2\pi\sqrt{x/a}$.

4. Conditions necessary for the simple harmonic motion of a particle. In simple harmonic motion the acceleration at any point is always towards the centre of motion and is directly proportional in magnitude to the distance of the point from the centre. Hence, in order that a particle may be capable of being set in simple harmonic motion it must be so fixed in its position of rest that, when the body suffers displacement, the force tending to restore it to its original position is in the line of the displacement and directly proportional to the displacement.

Fig. 6.

The following experiment illustrates the conditions under which a particle may move in simple harmonic motion.

Exp. 1. Suspend a body, M (Fig. 6), by a rubber cord or *light* spiral spring, S, strong enough to be stretched to about half its elastic limit by the weight of the body. When at rest the body is in equilibrium under the action of its weight and the equal and opposite tension in the cord.

If, however, the body be slightly displaced downwards or upwards in a vertical line the force resulting from this displacement, and

tending to restore the body to its original position, is in the line of displacement and directly proportional to the displacement. Hence, if the body is given a small vertical displacement and released it moves up and down in a vertical line, AB, in simple harmonic motion. This motion is easily set up and observed and the period may be determined by direct observation.

In the experiment described above the body M is in equilibrium under the action of two forces, the weight of the body and the tension in the cord. If the body suffers a vertical displacement the force tending to restore the body to its position of rest is therefore the difference, set up by the displacement, between the weight and the tension. Let the force which produces unit extension of the cord be denoted by w. Then, when the body is displaced vertically downwards through a small distance x the tension in the cord becomes greater than the weight of the body by $w\,x$ units. Also, when the body is displaced vertically upwards through a small distance x the tension in the cord becomes *less* than the weight of the body by $w\,x$ units. Hence, whether the small displacement x be upwards or downwards, the force acting on the body and tending to restore it to its original position of rest is equal to $w\,x$ and acts in the line of displacement.

The period of the motion of the body in this case is readily found by applying the relation $t = 2\pi\sqrt{x/a}$. The mass of the cord or spring is supposed to be small enough to be neglected in comparison with the mass of the suspended body. Hence in what follows the influence of the mass and weight of the cord is neglected.

Let m denote the mass of the body, then, since the force resulting from a displacement x is $w\,x$, the acceleration produced is given by $a = w\,x/m$. But if l denote the extension of the cord caused by the weight of M, then $mg = wl$, that is, $w = m\,g/l$. Substituting this for the value obtained for a above we get $a = g\,x/l$, and $\sqrt{x/a}$ is therefore equal to $\sqrt{l/g}$. That is, the period of the motion of the body is $2\pi\sqrt{l/g}$.

This result is readily verified by experiment: for example, in an experiment similar to that described above, the weight of the mass M was found to stretch the spring 11 cms., and the period of vibration was found by observation to be 2·13 secs. The period given by the formula $t = 2\pi\sqrt{l/g}$ is 2·10 secs., which, allowing for experimental errors and the effect of the mass of the spring, is in satisfactory agreement with the experimental result.

When a body moves in simple harmonic motion, as in this example, it is commonly said to *vibrate* or *oscillate*, and simple harmonic motion is considered as a special case of *vibratory motion* or *motion of vibration*.

5. Isochronism. The period of vibration in a given case of simple harmonic motion is constant and quite independent of the amplitude of vibration. It has been shown above that the period in any given case is given by $t = 2\pi\sqrt{x/a}$. The value of t therefore depends *only* upon the ratio x/a; that is, the period depends upon the *ratio* of the displacement to the force resulting from the displacement and not upon the magnitude of the displacement itself. In the example described above the period depends only upon the weight of M and the elastic strength of the cord or spring (the conditions which determine the ratio x/a) and is quite independent of the amplitude of the vibration. It follows from this that although a body in simple harmonic vibration when left to itself gradually loses energy and the amplitude of its vibration slowly decreases to zero value, the period of its vibration remains quite constant for all amplitudes until the body comes to rest. This property of vibratory motion is known as *isochronism*.

EXERCISES I.

1. Describe what is meant by *simple harmonic motion*. Define the *period* and *amplitude* of a point moving in simple harmonic motion.

2. A particle of mass 10 grammes fastened to the end of a string 1 metre long is swung uniformly round a circle of radius equal to the length of the string in a period of 1 second. Find the acceleration of the particle and the force in the string.

3. Show that when a particle is moving in Simple Harmonic Motion at a distance $\frac{\sqrt{3}}{2}$ of its amplitude away from the central position its velocity is half its velocity in the central position.

4. A heavy particle is supported by a fine inextensible string. It is pulled aside from the vertical and then released (Simple Pendulum). Show that its motion is Simple Harmonic, and express its period in terms of the length of the string and the acceleration due to gravity. Set up such a simple pendulum by suspending a small leaden ball by a piece of thread. Time its oscillations and thus verify your formula. Make the length of the string 99·4 cms. What is the period now?

5. A heavy particle is supported by a spring. Show that when displaced in a vertical direction the particle vibrates in Simple Harmonic Motion.

6. Explain what is meant by *isochronism*.

CHAPTER II.

ELASTICITY.

6. Elasticity. Elasticity is the property of matter which enables it to resist any strain involving change of size or change of shape, and by virtue of which it tends, when released after being strained in any way, to recover its original size and shape.

In the subject of elasticity the term *strain* applies to the change of configuration involving change of size or of shape, or of both size and shape combined, and the term *stress* applies to the force causing the strain. The term stress may also be used in speaking of the stress or stresses set up in the strained material as the result of the strain.

7. Limits of elasticity. When a body is subjected to a small enough strain it recovers completely its original form and size when the stress causing the strain is removed. If, however, the strain exceeds a certain limit, which is different for different materials, the body does not completely recover its original form and size when the stress is removed, but suffers a *permanent strain.* The amount of this permanent strain depends upon the extent to which the limit for perfect recovery is exceeded. If the limit is only slightly exceeded the permanent strain is small and the recovery nearly complete, but if the limit is greatly exceeded there is little or no recovery when the stress is removed and the greater part of the strain produced remains as a permanent strain. The limits within which perfect recovery from a particular form of strain takes place in any given material are known as the *limits of elasticity* for that form of strain in the material.

8. Moduli of elasticity. When the stress and strain are appropriately measured in any case of strain it is found that, *within the limits of elasticity*, the stress is directly proportional to the strain, that is, the ratio *stress/strain* is constant. When, however, the limits of elasticity are exceeded the strain increases much more rapidly than the stress and the ratio stress/strain rapidly decreases.

For any particular form of strain in a given material the value of the constant ratio of stress to strain within the limits of elasticity is known as the *modulus of elasticity* for that particular form of strain in the material. This modulus may be said to measure the elasticity of the material for the particular strain considered. A material may thus have a number of moduli of elasticity, each modulus corresponding to a particular form of strain. Two moduli, the modulus of bulk or volume elasticity and the modulus of form elasticity are independent and fundamental, and serve to specify two important properties of matter, *compressibility* and *simple rigidity*. Other moduli, such as *Young's modulus*, referred to later, are, however, all derived from these two fundamental moduli, and the relations between them can be determined when the strains to which they refer are properly defined.

All materials are more or less elastic, but wide differences are shown, not only in the elastic strength of the materials as measured by their moduli of elasticity, but also in the elastic limits for the different materials. Thus steel and glass are substances of very high elastic strength with very narrow limits of elasticity. Indiarubber, on the other hand, is a substance of low elastic strength with very wide limits of elasticity.

9. Modulus of volume elasticity. Two moduli of elasticity which are of importance in relation to sound are the modulus of volume, or bulk elasticity, and Young's modulus of stretching.

The *modulus of volume elasticity* is given by the ratio of the stress to the strain for a strain which involves change of volume without change of shape. For example, if a sphere of any material be subjected to uniform hydrostatic

pressure over its surface, any change in this pressure will produce a corresponding change of volume, but the form will remain that of a sphere. Similarly, with a body of any form subject to uniform hydrostatic pressure the effect of any increase or decrease of the pressure will be to cause a corresponding increase or decrease in volume without change of form. In this case the *stress* is measured by the change in pressure per unit area which causes the strain, and the *strain* by the ratio of the change in volume to the initial volume of the body. Thus, if p denote the increase of pressure, v the decrease in volume, and V the initial volume, then the strain is denoted by v/V, the stress by p, and the modulus of volume elasticity by the ratio $\frac{p}{v/V}$ or pV/v. That is, if k denote the modulus of volume elasticity we have, in the notation given,

$$k = pV/v.$$

10. Young's modulus of stretching. *Young's modulus* is the modulus of stretching strain. If a wire of length L is stretched by an amount l, then l/L measures the stretching *strain*, and if the stretching force is W and the area of cross-section of the wire a, then W/a is the *stress* applied. Hence, the modulus being, as usual, the ratio of stress to strain, Young's modulus in this case is evidently given by $(W/a)/(l/L)$ or WL/al. That is, if M denote Young's modulus we have, in the notation given,

$$M = WL/al.$$

11. Modulus of volume elasticity in the case of a gas. If a volume of gas be subjected to change of volume *without change of temperature*, then Boyle's law may be applied and the modulus of volume elasticity easily determined.

Thus, let P and V denote the pressure and volume of a quantity of a gas, and let the pressure be increased by an amount p, thereby producing a decrease in volume, denoted by v, without change of temperature. Here, therefore, the stress being denoted by p and the strain by v/V, the modulus of volume elasticity is evidently given by $k = pV/v$. But, as the initial pressure and volume are

denoted by P and V, and the final pressure and volume by $(P+p)$ and $(V-v)$, and as the temperature is supposed to be constant, Boyle's law gives the relation

$$PV = (P+p)(V-v),$$

or, neglecting pv as the product of two small quantities,

$$PV = PV + Vp - Pv.$$

That is, $$Vp = Pv, \text{ or } V/v = P/p.$$

Substituting this value of V/v in the value just obtained for the modulus of volume elasticity, we get

$$k = pV/v = pP/p = P.$$

That is, when the volume strain in a gas takes place without change of temperature, the modulus of volume elasticity is measured by the pressure of the gas.

12. Isothermal and adiabatic elasticity. When a substance is subjected to a strain of any kind the work done in producing the strain is converted partly into potential energy of strain in the substance and partly into heat in the substance in the region where the strain exists. In some cases, as in the compression of a spiral spring for example, the work is practically all converted into potential energy of strain, and in other cases, as in the compression of a gas, the work is practically all converted into heat.

The molecular constitution of a gas is such that, although it may be subjected to compression or expansion, it cannot (to any appreciable extent) acquire potential energy of strain. Hence, when a gas is compressed the work done in compressing it is not stored up in the gas as potential energy of strain, but appears as heat in the gas and its surroundings. Similarly, when a gas under compression is allowed to expand, the energy necessary to effect the expansion against external pressure is not derived from a store of potential energy of strain in the gas, but from the heat in the gas and its surroundings. Hence, if a gas is subjected to compression or expansion under conditions which prevent the passage of heat between the gas and its surroundings, that is under *adiabatic* conditions, the heat resulting from compression is produced in the gas, and the heat required during expansion is derived from the gas. That

is, under adiabatic conditions, compression of a gas is accompanied by a rise of temperature in the gas, and expansion is accompanied by a fall of temperature.

When a gas is subjected to a *sudden* compression or rarefaction the conditions of the strain are practically adiabatic. A gas is a very bad conductor of heat, so that when subjected to a very sudden compression, the heat produced by the compression at any point is not conducted away to any appreciable extent from that point during the short time of the strain, and a sudden rise of temperature throughout the region of compression is the result. Similarly, when a gas undergoes rapid rarefaction, heat is suddenly absorbed from the region of rarefaction and, as practically no heat can be conducted to the region from its surroundings during the short time of the strain, a sudden fall of temperature throughout the region of rarefaction is the result.

If, however, a gas is subjected to compression or rarefaction, under conditions such that heat may pass readily into or out of it, then the operation may be conducted without any change of temperature. For example, if a gas is compressed *very slowly* in a copper cylinder maintained at a constant temperature—by an ice jacket, say—the heat resulting from the compression passes as it is produced from the gas to the cylinder without producing any rise of temperature. Similarly, when the gas expands very slowly in the cylinder the heat absorbed from the gas to do the work of expansion is supplied to it by conduction through the cylinder. An operation conducted in this way *at constant temperature* is called an *isothermal* operation.

It is evident from what has been said that the value of the modulus of volume elasticity for a gas must depend upon whether the strain is effected under *adiabatic* or *isothermal* conditions.

When a gas is compressed under adiabatic conditions the heating resulting from the compression produces expansion which opposes the compression, and so a greater stress is necessary to produce a given strain than would be required to produce the same strain under isothermal conditions. Also, when a gas expands under adiabatic conditions, the cooling resulting from the expansion

produces contraction which opposes the expansion, and so a greater stress is necessary to produce a given strain than would be required to produce the same strain under isothermal conditions. Hence the elasticity under adiabatic conditions, that is, the *adiabatic elasticity*, is greater than the elasticity under isothermal conditions or the *isothermal elasticity*.

It has been shown above that under isothermal conditions the elasticity of a gas is measured by its pressure. It can further be shown that under adiabatic conditions the elasticity of a gas is measured by γ times its pressure, γ being the ratio of the two specific heats of the gas; that is, if P denote the pressure of a gas, then the isothermal elasticity of the gas is denoted by P and the adiabatic elasticity by γ P. The value of γ for air is about 1·41.

EXERCISES II.

1. Define *limits of elasticity*, *stress*, *strain*, *modulus of elasticity*.

2. A certain mass of air occupies 101 c.cms. at a pressure of 1,000,000 dynes per sq. cm. The pressure is increased slowly to 1,010,000 dynes per sq. cm. and it is found that the volume becomes 100 c.cms. Find the isothermal elasticity.

3. Tie strings to the ends of a thick short cord of indiarubber, say 20 cms. long and 1 cm. in diameter. Tie one string to a scale pan and the other to a nail so that the cord hangs vertically. Mark with ink 2 points on the cord between but near the points of attachment of the string. Measure the distance between the points with a mm. scale and the diameter of the cord with a pair of vernier calipers. Place weights in the scale pan and for each weight find the increase in length of the portion of cord between the ink marks. Find the average value of (total weight ÷ total elongation) and calculate the value of Young's modulus for indiarubber.

4. An indiarubber rod 10 inches long and of circular cross section of area ·5 sq. inch is found to be stretched 1 inch by a weight of 25 pounds. Find the Young's modulus in lbs.-wt. per sq. inch.

5. Show that if the area of cross section of a rod is unity and remains so as the rod is pulled out, the Young's modulus of the substance is numerically equal to the force which would double the length of the rod.

6. Under what conditions is the modulus of volume elasticity of air measured (*a*) by its pressure, (*b*) by 1·41 times its pressure?

CHAPTER III.

VIBRATORY MOTION.

13. Vibratory motion. Vibratory motion is always of a *periodic* character, but is not generally of the simple harmonic type. Simple harmonic motion is the simplest type of periodic motion, and it can be shown by *Fourier's theorem* that any periodic motion is the resultant of a number of simple harmonic components. The number of components may be large or small, and is sometimes infinite.

A body may therefore, under certain conditions, vibrate in simple harmonic motion, but more generally vibrates in a non-harmonic but periodic mode compounded of a number of simple harmonic components. It will be learnt later that a body can generally vibrate in several simple harmonic modes of different periods, and that its general mode of vibration is a resultant mode compounded of some or all of these harmonic modes as components.

14. Phase. When a particle vibrates in simple harmonic motion it moves from point to point in its path, having a definite velocity and direction of motion at each point. The *phase* of the particle at any instant is determined by its position and direction of motion in the path of vibration at that instant. Thus, if the body vibrates along the path A O B (Fig. 7), its phase at any instant may be indicated by stating that it is at a particular point X in its path of vibration and moving in the direction O B.

Fig. 7.

Obviously the mere statement of position at X is not sufficient to specify the phase, for at the point X the particle may be moving either in the direction O B or [illegible]. When a vibrating body, starting from any point [illegible] path, continues its motion until it again passes through the

same point in the same direction, that is, until it is again in the same phase, it is said to perform a *complete vibration*. Thus, starting from *any point* X in the path A O B, the particle, in moving on from X to B, then back from B through O to A, and then from A back through O to X, performs a complete vibration. The time of performing a complete vibration is called the *period* of the vibration. It is also obviously the time in which the same phase of vibration periodically recurs.

In order to specify phase conveniently it is usual to take a definite starting point to determine complete vibrations and then to specify the phase at any instant by stating the interval of time, as a fraction of a period, at which this instant follows the beginning of a complete vibration. Thus, in Fig. 7, if complete vibrations are counted from the point O as starting point, with the body moving in the direction O B, the phase when the body is at B is the phase which occurs one-quarter of a period after the beginning of a complete vibration. Similarly, when the body is at O, moving in the direction O A, the phase is that which occurs half a period after the beginning of a complete vibration.

If two bodies are vibrating with exactly the same period and amplitude they may still differ in phase. This difference in phase is usually expressed as a fraction of the period of vibration. Thus one body may be a quarter or half, or any fraction of a period in advance of or behind the other. For example, if we have two simple pendulums vibrating with exactly the same period, but in such a way that when one is at the middle point of its swing and moving towards the right the other is at its extreme position on the right, then the former may be said to be a quarter of a period later in phase than the latter.

15. Frequency of vibration. When a body vibrates in periodic motion it has been shown that the motion consists of a succession of complete vibrations, each of which occupies a definite constant period of time called the *period* of the vibration. Another way of expressing this periodicity of the motion is to say that the body executes a definite

constant number (not necessarily integral) of complete vibrations per second. The number of complete vibrations made by the vibrating body in one second is known as the *frequency* of the vibration.

If the *frequency* of vibration for a vibrating body is denoted by n, then the body makes n complete vibrations per second and the *period* of its vibration is $1/n$ second. For example, if a body has a frequency of 256 vibrations per second, then its period of vibration is 1/256 second. Similarly, if the period of vibration of a vibrating body is t seconds, the frequency of vibration is $1/t$ vibrations per second. For example, if the period of vibration is ·005 second, the corresponding frequency is 200 vibrations per second. This relation between period and frequency of vibration is most concisely expressed by the relation $n\,t = 1$, where n denotes the frequency and t the period of the vibration.

16. Variation of the displacement of a particle in simple harmonic motion during a complete vibration. Imagine a particle to vibrate in simple harmonic motion along the line A O B (Fig. 8), and suppose the complete vibrations to begin when the particle is at O moving in the direction O A.

Fig. 8.

During the first quarter period the displacement increases from zero to its maximum value O A, and in the second quarter period it decreases from this maximum value to zero again. In the third quarter its direction is reversed and it increases from zero to a maximum value O B, and in the fourth quarter it decreases again from this maximum value to zero. Thus, if the amplitude or maximum displacement O A, or O B, be denoted by r the changes of the displacement during a complete vibration may be given as from 0 to r in the first quarter period, from r to 0 in the second quarter, from 0 to r again, but with the direction of displacement reversed, in the third quarter, and then from r to 0 in the fourth quarter.

It is important, however, to go further than this and to determine how the displacement increases and decreases during each quarter period.

Referring to Fig. 3, it will be seen that as the point P moves round the circle from C through A, D, and B back to C, the point p moves in simple harmonic motion along the path O A O B O and its distance from O, that is, the displacement O p is always equal to $r \sin \beta$, where β denotes the angle P O C, which increases from 0° to 360° during the complete vibration. That is, during a complete vibration, beginning at O, the displacement Op varies in the same way as the sine of an angle changes as the angle increases from 0° to 360°.

The value of the displacement at any instant during a complete vibration is thus represented by $r \sin \beta$, where r is the amplitude of the vibration, and β is the value of the angle P O C at the given instant and may be anything between 0° and 360°. If the angular velocity of the point P be ω, that is, if the period of the motion is $2\pi/\omega$, then the value of β at the end of any time t from the beginning of the motion is ωt and the displacement at this instant is given by $r \sin \omega t$. Since the angle β is described uniformly with time and changes from 0° to 360° in the period of vibration, any fraction of a period may be represented by the corresponding fraction of 360°.

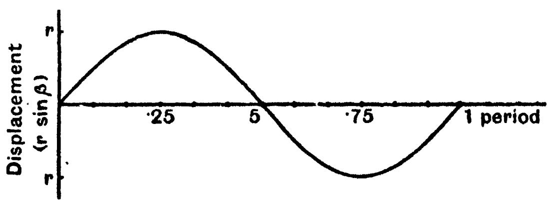


Fig. 9.

Hence if we plot a curve, as in Fig. 9, with values of $\beta/360$ or fractions of a period up to one whole period as abscissae, and $r \sin \beta$ as ordinates, the curve represents the variation of the displacement of the particle during a complete vibration. This curve, with its ordinates proportional to $\sin \beta$, is known as the *sine curve*.

A particle may vibrate in well defined periodic motion other than simple harmonic motion; in this case, however, the variation of its

displacement, with time, during a complete vibration will not follow the sine rule, and its curve of displacement curve will not be the sine curve but some other periodic curve characteristic of the motion.

The variation of displacement with time during a complete vibration is important as one of three features which serve to characterise and distinguish periodic motions. These three features are the *frequency*, the *amplitude*, and the *displacement variation* of the motion. These features are clearly exhibited by the displacement curve of the motion; the length of the curve indicates the period and therefore the frequency, the maximum ordinates indicate the amplitude, and the form of the curve indicates the character of the displacement variation.

17. Transformation and dissipation of energy during vibration. When a particle capable of vibration is displaced from its position of rest, work has to be done against the forces opposing displacement and the particle therefore gains potential energy during the displacement. At any point the gain in potential energy is measured by the work done against opposing forces in effecting the displacement from the position of rest up to that point.

When a particle in vibration is at either of the extreme points of its path it is, for an instant, at rest, and its *energy of vibration* may therefore be measured completely by the potential energy gained during the displacement of the particle from its normal position of rest to this extreme position, or by the work done in effecting this displacement against opposing forces. As, however, the particle passes in vibration from an extreme position towards the centre of its path it gradually loses potential energy and gains kinetic energy until, as it passes through the central position, its *energy of vibration* is wholly kinetic and may be expressed by the quantity $\frac{1}{2} m v^2$, where m denotes the mass of the particle and v its velocity as it passes through this position. If for the moment we neglect the dissipation of energy due to work done against friction during vibration, the kinetic energy of the particle in passing through its central position must be exactly equal

in magnitude to the potential energy which measures its energy of vibration in an extreme position. Similarly, as the particle in vibration passes from its position of rest towards an extreme position, it gradually loses kinetic energy and gains potential energy until on attaining an extreme position its energy of vibration is again wholly potential. At any point between an extreme position and the central position the energy of vibration of the particle is evidently partly kinetic and partly potential, the amount of the potential energy at any point being equal to the work done in displacing the particle from the central position up to that point. Further, if we neglect the dissipation of energy due to work done against friction, the sum of the potential and kinetic energies of the particle at any point must be *constant* and equal to the potential energy which measures the energy of vibration in an extreme position, or to the kinetic energy which measures the energy of vibration at the central position. This constant quantity of energy is the *energy of vibration* or *vibratory energy* of the particle, and is evidently, during the vibration of the particle, subject to periodic transformation from the potential to the kinetic form, and from the kinetic to the potential form.

The vibratory energy of a particle in vibration is constant during vibration only if there is no dissipation of energy by work done against *frictional* resistances opposing vibration. In any actual case of vibration, however, it is impossible to avoid friction. There is therefore always in actual practice a gradual dissipation of the energy of vibration by the work done against friction during vibration. As a result a particle in vibration gradually loses its energy of vibration, the amplitude of vibration becomes smaller and smaller, and finally, when the whole of the energy of vibration is dissipated as heat, the particle comes to rest.

EXERCISES III.

1. Define *phase* and *frequency* in relation to vibration.

2. The frequency of vibration of a vibrating body is 200. Find its period.

3. Two bodies are vibrating in Simple Harmonic Motion. Their periods are 10 and 20 seconds respectively; their amplitudes are 20 and 10 cms. respectively. Compare their average velocities.

4. Take a curve similar to that given in Fig. 9 as the displacement curve of a particle in vibratory motion and construct from it a figure showing the position of the particle in its path of vibration at intervals of one-twelfth of a period during a complete vibration.

5. Taking times as abscissae and displacements as ordinates plot a curve showing the successive positions of a particle vibrating in Simple Harmonic Motion during a complete vibration.

6. Two particles are vibrating in Simple Harmonic Motion of the same period along the same line. One is at the end of its path while the other is at the middle of its path. Find the difference in phase.

7. Calculate the phases of a harmonically vibrating body (1) when its velocity is only half its maximum velocity, (2) when its displacement is half its amplitude.

8. Show that the maximum velocity of a particle vibrating with Simple Harmonic Motion is equal to 1·57 times its average velocity.

9. Let t = the period of a harmonically vibrating point. Show that its velocity at any instant is $\frac{2\pi}{t}$ of the displacement which it will have a quarter of a period later, and that its acceleration at any instant is $\frac{2\pi}{t}$ of the velocity it will have a quarter of a period later.

10. Show that the energy of a body vibrating with Simple Harmonic Motion is sometimes wholly potential, sometimes wholly kinetic.

11. A particle moves with a motion compounded of a Simple Harmonic Motion East and West and a Simple Harmonic Motion North and South; these motions have the same period and amplitude. Plot its path

(1) when there is no difference in phase;
(2), (3), (4) when the difference of phase is $\frac{1}{4}$, $\frac{1}{2}$, and $\frac{3}{4}$ respectively.

12. If the amplitudes of the two motions of the last question are unequal, plot the path when the difference of phase is $\frac{1}{4}$.

CHAPTER IV.

VIBRATION OF BODIES.

18. Vibration of elastic bodies. When an elastic body, such as a rod of any elastic material, or an elastic system, such as a spiral spring, is strained in any way within its limits of elasticity the stress is known by experiment to be proportional to the strain.

This simple relation between stress and strain is usually known as **Hooke's law**.

Exp. 2. Suspend an indiarubber cord or spiral spring as in Exp. 1, and attach a scale pan to the lower end. The cord (or spring) can now be stretched by placing suitable weights in the scale pan, and the stretching strain can be determined by measuring with a scale the increase in the distance between two marked points on the cord. It will be found that, within the limits of elasticity, the stretching strain produced is directly proportional to the stretching stress applied.

Exp. 3. Fix a uniform lath or strip of wood or other material with a G-clamp to the top of a table so that the greater part of the strip projects horizontally beyond the edge of the table, and suspend a scale pan from the free end of the strip. The strip can now be bent by placing suitable weights in the scale pan, and the bending can be determined by measuring the vertical deflection of the end of the strip. It will be found that within the limits of elasticity the bending produced is directly proportional to the weights applied.

Hooke's law evidently implies that when an elastic body or system is strained, the stress set up in the body, as the result of the displacement which constitutes the strain, is directly proportional to that displacement. Hence, when the strained body is suddenly released from the constraining stress, the conditions are such that the body may be set in vibratory motion.

19. Transverse vibration of a rod. Let a rod of wood or metal or any elastic substance be fixed in a vertical position with the lower end clipped in a vice and the upper end free, as shown at *a* in Fig. 10.

If the rod is now bent from its stationary position at *a* into the position shown at *b* and then released, it at once flies back through its initial position *a* to a position *c* on the other side of *a*, and so continues to vibrate backwards and forwards on each side of *a* with gradually decreasing amplitude until it finally comes to rest in its initial position.

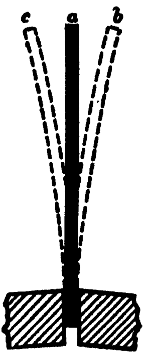


Fig. 10.

At all points in the rod the direction of vibration of its particles is at right angles to, or transverse to the length of the rod. For this reason the mode of vibration here considered is called *transverse* vibration.

It will be noticed that the amplitude of vibration is greatest at the free end of the rod and decreases towards the fixed end, where it is necessarily zero. At the free end the initial amplitude is the distance *a b*, but as the vibration goes on the amplitude here, and at all points in the rod, decreases gradually until the vibration finally ceases. Throughout the vibration the free end of the rod is the point of maximum amplitude of vibration, while the fixed end is necessarily a point of zero amplitude. On the other hand, however, the fixed end is the point where maximum bending strains occur and the free end is the point of zero strain.

The fixed end is here a *node* in the vibrating rod, a node being a point in a vibrating body at which the amplitude is zero and the strain a maximum. The free end, being opposite in character to a node, is usually called an *antinode*, an antinode being a point in a vibrating body at which the amplitude is a maximum and the strain zero.

In a *complete vibration* the rod, starting, say, from the position *a*, moves up to *b*, then back to *c*, and then to *a*

again, and its continued vibration is evidently a periodic repetition of complete vibrations. The time of describing a complete vibration is the *period* of vibration of the rod, and the number of complete vibrations per second is the *frequency* of its vibration. Since the period of vibration is not affected by the amplitude, the frequency of vibration of the rod is constant and does not change as the vibration set up by plucking the rod dies away to rest.

The calculation of the period of vibration of the rod from the relation $t = 2\pi\sqrt{x/a}$ requires methods which we cannot here apply. It should, however, be noticed that when, for a small value of x, the value of a is great, the period t is very small. That is, when the rod is stiff and difficult to bend so that the force resulting from a small displacement is great, then the period of vibration of the rod is small and the frequency high. For the same reason the frequency of vibration for a particular rod may be increased by decreasing the length of the portion set in vibration, for the shorter this length is the more difficult it is to bend the rod.

20. Longitudinal vibration of a rod. Let the rod be fixed in a vice as described in the preceding article.

If the rod be now stretched so that the end at a (Fig. 11) is displaced to b and then released it at once recovers its original length, and, by virtue of the kinetic energy it has gained during this recovery, again acquires potential energy of strain by shortening until the free end reaches a point c, such that ac is *nearly* equal to ab. The rod then recovers its original length, lengthens by an amount *nearly* equal to ac, and so continues to vibrate up and down, shortening and lengthening so that the free end oscillates above and below its initial position at a, with gradually decreasing amplitude until the vibration dies away and the rod is at rest.

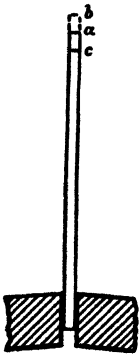


Fig. 11.

In this case the vibration at all points in the rod is

along its length, and the mode of vibration is therefore called *longitudinal vibration.*

As in the case of transverse vibration the fixed end of the rod is a *node* and the free end an *antinode.*

The mode of vibration of the rod is comparatively simple. The rod in vibrating lengthens and shortens periodically so that each point in its length vibrates, in the same period and in the same phase, up and down, above and below its normal position. The amplitude of vibration is greatest at the free end, and diminishes along the rod towards the fixed end where it is zero.

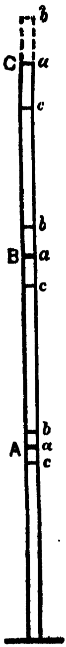


Fig. 12.

In the case of the transverse vibration of a rod the amplitude of vibration at any point in the length of the rod is determined by the form of the flexural curve which the bent rod takes up in the positions b and c, and its general value at any point cannot here be discussed.

In this case of longitudinal vibration, however, the amplitude of vibration at any point in the length of the rod can be expressed in terms of the maximum amplitude at the free end. It will be shown later that the amplitude increases along the length of the rod from the fixed end to the free end in the same way as the sine of an angle increases as the angle increases from 0° to 90°. So that if l denote the length of the rod, then the amplitude at any point, at a distance x from the fixed end, is given by $r \sin\left(\frac{x}{l} \cdot \frac{\pi}{2}\right)$, where r is the amplitude at the free end. Fig. 12 shows roughly to scale the relative amplitudes of vibration at points A, B, and C, one-third, two-thirds, and three-thirds of its length from the fixed end of a rod in longitudinal vibration. As the rod describes a complete vibration, starting, say, from its normal length and about to lengthen, the points at A, B, and C *simultaneously* start from the a positions, move up to the b positions, then back through the normal positions to the c positions, and finally back again to the a positions. The amplitudes of vibration at A, B, and C are evidently proportional to the sines of 30°, 60°, and 90°. In the figure the relative magnitudes of these amplitudes are correctly indicated, but their actual magnitudes in proportion to the length of the rod are greatly exaggerated. If a curve be drawn, as in Fig. 13, with distances along the rod from the fixed end as abscissae and $r \sin\left(\frac{x}{l} \cdot \frac{\pi}{2}\right)$ as ordinates, the curve represents how

the amplitude of vibration increases from the fixed end to the free end of the rod.

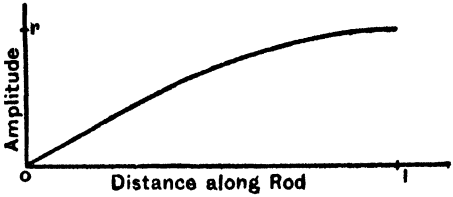


Fig. 13.

In the case of a rod of wood, metal, glass, or any similar material, the force necessary to produce even a very small elongation is extremely great. For this reason the frequency of vibration of a rod in longitudinal vibration is very high. Notwithstanding the great force necessary to extend a rod, longitudinal vibration may be easily set up in a rod of wood or metal by gripping it lightly with a resined rubber and drawing the rubber smartly along its length. Similarly vibration may be set up in a glass rod or tube by using a pad of cotton wool wet with alcohol as a rubber.

21. Strains produced in a rod in longitudinal vibration. It has been stated above that when the rod is in longitudinal vibration the fixed end is a node and the free end an antinode. This implies that the strain in the rod is a maximum at the fixed end and zero at the free end. It may not, however, be clear what is the exact nature of the strain set up in the rod during vibration or why this strain should be a maximum at the fixed end. During vibration the rod successively lengthens and shortens. The strain set up is therefore one of extension or compression. Let $a\,b\,c\,d$ (Fig. 14) represent a *thin* transverse slice of the rod taken anywhere in its length between the two plane sections ab and cd. Now the strain of compression or extension which may be set up in the slice $a\,b\,c\,d$ is

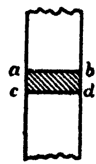


Fig. 14.

evidently determined, not by the actual displacement of the sections *a b* and *c d*, but by the *difference* of their displacements. For example, if both sections are displaced upwards and *a b* is displaced further than *c d*, then the slice *a b c d* must suffer an *extension* equal to the difference of these two displacements. Similarly, if both sections are displaced downwards and *a b* is again displaced further than *c d*, then the slice *a b c d* suffers a compression equal to the difference of these two displacements. If *d* denote the difference of the displacements and D the thickness of the slice, then *d*/D measures the *strain* in the slice, for *d* measures the change in the thickness or volume of the slice in the same way as D may measure its initial thickness or volume.

It is evident therefore that the maximum strains to which a slice such as *a b c d* is subjected at any point in the rod depend upon the difference of the amplitudes of vibration of the sections *a b* and *c d*. Now in Fig. 13 we have represented to scale the amplitude of vibration, that is, the extreme displacement, at any point in the length of the rod, and if we take two sections, such as *a b* and *c d*, close together, somewhere very near the free end of the rod, it is clear from the form of the curve of amplitudes that the *difference* of their displacements will be very small, even when these displacements are greatest, and the slice *a b c d* will therefore suffer very little strain either of extension or compression. It is also clear from the curve that for two sections a short fixed distance apart the difference of their amplitudes increases rapidly as the fixed end of the rod is approached, and the maximum strain set up in any slice between the sections increases as the distance of the slice from the fixed end decreases. It follows from this that if we suppose the rod divided into a large number of thin transverse slices the extreme strain in the slice at the free end is practically zero ; in the slice at the fixed end it is a maximum ; and, for intermediate slices, the extreme strain increases gradually from the free end to the fixed end.

During a complete vibration of the rod the cycle of strain is exactly the same in character, but different in range, for all the slices. Starting from the normal state and supposing the rod to lengthen, then recover its original length, then shorten, and finally recover its original length again, each slice first undergoes a gradually increasing extension, then recovers its original state, then undergoes a gradually increasing compression, and finally recovers its original state again. The *range* of this cycle, however, is greatest at the node or fixed end and diminishes to zero at the antinode or free end.

22. Longitudinal vibration of a column of air. A column of air, or other gas, enclosed in a tube closed at one end and open at the other, may be made to vibrate longitudinally in exactly the same way as the rod described in the preceding article. The air at the closed end of the tube is the fixed end of the column, and the air at the open end is the free end, so that the column vibrates with a node at the closed end and an antinode at the open end.

The mode of vibration of the column is exactly similar in detail to that of the rod. A column of air or gas is not, however, capable of extension or stretching in the same sense as a rod of a solid. A layer of gas at any point in the column undergoes *expansion* or *rarefaction* under the conditions which would produce extension of a slice of a solid rod. The strains in the column during vibration therefore consist of compressions and rarefactions. A compression at any point necessarily involves an increase of density and an increase of pressure at that point. Similarly, a rarefaction at any point implies a decrease of density and a decrease of pressure at that point.

During the vibration of the column the node is at the closed end of the tube. This node is the region of maximum compression and rarefaction, that is, of maximum change of density and therefore of maximum change of pressure. The antinode on the other hand, being the region of no strain, is the point in the column at which there are no changes of density and no changes of pressure. This is consistent with the fact that the antinode, being at the open end of the tube and in free communication with the outer air, is a point at which changes of density or of pressure could not be set up.

It is important to notice that the air at the closed end of the tube, being fixed, *must* be a node, and the air at the open end, being in free communication with the outer air, *must* be an antinode. The column of air must therefore vibrate in such a way that the closed end is a node and the open end an antinode. The mode of vibration described above is the *simplest* which satisfies these conditions.

23. Transverse vibration of a flexible string. Let a string, cord, or thin wire, AB (Fig. 15), be stretched tightly between the fixed points A B.

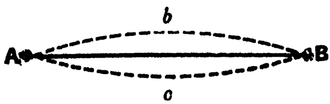


Fig. 15.

If it is plucked aside into the position A *b* B or A *c* B and released the string vibrates rapidly backwards and forwards between these positions with gradually decreasing amplitude until it comes to rest.

In this case the vibration is due mainly to the tension in the string and not to the elasticity of its material. When the string is deflected into one of the lateral curved positions the action of the tension tends to restore it to its initial straight position and opposes its displacement from this position. For small lateral displacements the force resulting from displacement is proportional to the displacement, and the conditions are therefore such that the string is set in *transverse* vibration, and each point on it vibrates at right angles to the length of the string.

It will be noticed that the two fixed ends of the string at A and B are *nodes*, and as such are places of no amplitude of vibration but of maximum strain. As the points are fixed the amplitude is necessarily zero. The strain at any point in the string is, in this case, the change in the direction of the string (and therefore of the tension in the string) at that point, as measured by the angle which the tangent to the string at that point makes with its initial direction. It is evident from Fig. 15 that strain as thus defined is a maximum at the points A and B. Evidently also the vibrating string has an antinode at its middle point. The amplitude is here a maximum and the strain zero, for at the middle point the direction of the string is always parallel to its initial direction.

In this case, as in others previously considered, the amplitude of vibration increases from zero at a node to its maximum value at the antinode. The law of its variation with distance along the string is in an actual case complicated by the flexural rigidity of the string, but if this rigidity be assumed negligible the sine law described in Art. **20** applies. That is, with a perfectly flexible string the amplitude at any point in the string at a distance x from a node is $r \sin\left(\frac{x}{l} \cdot \frac{\pi}{2}\right)$, where r is the amplitude at the antinode and l the distance from node to antinode or half the length of the string.

24. Forced vibration. If a periodic force is applied to a body or system capable of vibration so as to tend to set it in vibration, then the body or system will ultimately vibrate in the same period as that of the applied force whatever may be the natural period of the body or system. This is the general principle of *forced vibration*. When the natural period of vibration of the body or system is the same as that of the applied force, then the forced vibration is readily and quickly set up. This particular case of the general principle of forced vibration is generally known as *sympathetic vibration* or **resonance**.

This principle may be conveniently illustrated by the forced vibration of simple pendulums of different length under the action of a periodic force derived from the vibration of another simple pendulum. Let four simple pendulums, A, B, C, and D, be suspended from a light wood lath fixed horizontally with its "width" in a vertical plane, as shown in Fig. 16, in which the plane of vibration of the pendulums is supposed to be at right angles to the plane of the paper. Let A be the actuating pendulum with a rather heavy bob, B a pendulum of the same length as A, C a slightly longer, and D a slightly shorter pendulum. It will be found that when the pendulum A is set in vibration the lath is set in forced transverse vibration of small amplitude, but of the same period as the pendulum by the periodic force applied to it at the point of attachment of the swinging pendulum. As a result of the vibration of the lath a periodic force of the same period as that of pendulum A is applied at the

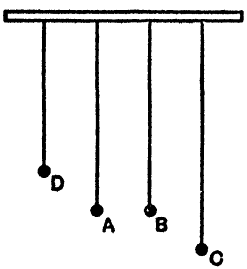


Fig. 16.

points of attachment to each of the pendulums B, C, and D. The application of this periodic force tends to set the pendulums in vibration, and it will be found that the pendulum B of the same length, and therefore of the same natural period, as A is quickly set in vibration. This is an instance of the particular case of forced vibration known as *resonance*. The pendulums C and D, however, first get up a slight swing, then come to rest and continue this process for some little time, but ultimately they settle down into steady vibration, with exactly the same period as that of the pendulum A. It will be found on examination that the mode of vibration of C is that shown in Fig. 17, while that of D is shown in Fig. 18.

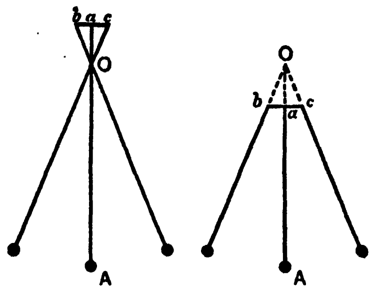


Fig. 17. Fig. 18.

In each case it will be seen that as the point of attachment of the pendulum, *a*, vibrates in the path *b c* (which may be *very short*) with a period equal to that of the pendulum A the suspension thread adjusts itself so that each pendulum vibrates as a pendulum of length, O A, equal to that of A and with the same period as A. It will be noticed also that the amplitude of vibration produced in C and D is less than in the case of A, and that while the pendulums A, B, and C are in the same phase, D is in the opposite phase or differs from them in phase by half a period.

Forced vibrations are often set up, not in a body as a whole, but in the particles of the body. For example, if a steel strip is clamped in a vice and set in vibration by plucking it, as described in Art. 19, forced vibrations are set up in the particles of the iron or wood vice in which the strip is clamped. Similarly, if a wire is stretched between two wood pegs on a table, the vibration of the wire sets up

forced vibrations in the adjacent particles of the pegs and table top. Forced vibrations of this character are more readily set up in wood than in any other substance.

25. Energy and vibration. When a substance is subjected to a strain work has to be done against the elastic stresses which oppose the strain and also, to some extent, against internal molecular friction in the substance. The work done against internal friction is, in elastic substances, comparatively small and is entirely dissipated as heat in the substance. The work done against opposing elastic stresses is transformed partly into potential energy of strain, and partly into heat, in the strained substance.

When the stress to which a strain is due is removed the substance is free to recover from strain, and the potential energy of strain in the substance, together with a quantity of heat equal to that produced by the work done against elastic force in effecting the strain, is available for expenditure. This energy is expended in giving the displaced portions of the strained substance kinetic energy as they return to their initial positions and to a small extent in again doing work against internal friction. Hence, when the substance regains its unstrained condition, the portions originally displaced possess an amount of kinetic energy equal to the potential energy and heat due to the strain, diminished only by the energy dissipated as heat during the recovery from strain. This kinetic energy will evidently be expended in causing a reversal of the initial strain, that is, the displaced portions of the body, by virtue of the kinetic energy they possess, pass beyond their initial positions of rest, and a strain similar to that first produced, but with the direction of its displacements reversed, is set up. The whole of the kinetic energy of the displaced portions, with the exception of a small amount again dissipated as heat in doing work against molecular friction, is thus retransformed into potential energy of strain. When this retransformation is complete, recovery from the strain set up at once begins, and the processes of transformation of the potential energy of strain into

kinetic energy and the retransformation of kinetic energy into potential energy of strain, accompanied by gradual dissipation of energy as heat, are repeated, vibration after vibration, until the whole of the initial energy is dissipated as heat.

From what has been said it will be seen that vibration may be looked upon as the process by which a strained body loses by dissipation the potential energy of strain which it possesses. When the body is first strained a certain quantity of strain energy is supplied to it. If the body is free to dissipate this energy it at once proceeds to do so by vibration. At each vibration the energy in the body undergoes transformation from potential to kinetic energy, and from kinetic energy to potential energy, and also suffers a small loss by dissipation on account of the work done against frictional resistances. The energy in the body therefore becomes less and less as the process of vibration goes on, and the body finally ceases to vibrate when the whole of the initial energy it possessed is lost by dissipation during the vibration. If the frictional resistances opposing vibration are great the loss by dissipation at each vibration will be considerable, and the body will cease to vibrate after executing comparatively few vibrations. If, however, the frictional resistances are small the loss of energy at each vibration by dissipation will be small and the body may execute a very large number of vibrations before it comes to rest.

26. The energy of a vibrating body. It is beyond the scope of this book to establish a general expression for the energy of a vibrating body. It is important, however, to know that the energy of a vibrating particle is proportional to the square of its amplitude of vibration. This result is readily obtained. Let r denote the amplitude of vibration, then, since Hooke's law applies, the force opposing the displacement of the particle from its initial position to its extreme position increases *uniformly* from o to $k\,r$, where k is a constant, and its mean value is therefore $\frac{1}{2}k\,r$. The work done during a displacement of amplitude r is therefore given by $\frac{1}{2}k\,r\,.\,r$ or $\frac{1}{2}k\,r^2$. The potential energy of the

particle in its extreme position, when all the energy it possesses is potential energy, is equal to this work, $\frac{1}{2} k r^2$, and the energy of the vibrating particle is therefore proportional to r^2, the square of its amplitude of vibration.

The energy of the vibrating particle may also be expressed in terms of the mass of the particle. If v denote the velocity of the particle when its displacement is zero and when all its energy is *kinetic energy*, then the energy of the particle is given by $\frac{1}{2} m v^2$ where m denotes the mass of the particle. But, as shown in Art. 2, $v = r\omega$ and $\omega = 2\pi/t$, therefore we get $v^2 = (r\omega)^2$ or $(2\pi r/t)^2$, and the energy of the vibrating particle is given by $\frac{1}{2} m \cdot \left(\frac{2\pi r}{t}\right)^2$ or $2\pi^2 m r^2/t^2$ or $2\pi^2 m r^2 n^2$, where n is the frequency of vibration. This is an expression for the energy of a particle of mass m vibrating with amplitude r and period t or frequency n.

A vibrating body may be supposed to be made up of an infinite number of particles of mass m, all vibrating with the same period, but with different amplitudes. The energy of a vibrating body may therefore be expressed as $2\pi^2 n^2 \Sigma(m r^2)$ or $2\pi^2 n^2 \mathrm{M}\, \bar{r}^2$, where M denotes the mass of the body and $\mathrm{M}\,\bar{r}^2 = \Sigma(m r^2)$.

EXERCISES IV.

1. Describe the transverse vibration of a rod fixed at one end. Where is the node and where the antinode? What are the properties of the node and antinode?

2. Why does a steel knitting-needle fixed in a clamp at one end vibrate with greater frequency than a wooden rod of the same size fixed in the same way?

3. Describe an experiment to show that the frequencies of transverse vibration of rods increase as the lengths of the rods diminish.

4. Describe the longitudinal vibration of a rod fixed at one end.

5. What is the formula connecting the amplitude of any point of a longitudinal vibrating rod of length, l, with the amplitude, r, at the free end, and its distance, x, from the fixed end? Can you give any reasons for supposing that this formula may be true?

6. Describe an experiment you would perform to set a glass rod in longitudinal vibration.

7. Show that the strain in a longitudinally vibrating rod fixed at the middle is a maximum near the middle and a minimum at the ends.

8. A column of air in a tube closed at one end is in longitudinal vibration. Why must there be a node at the closed end?

9. Explain *forced vibration* and *resonance*. Describe experiments to exhibit them.

10. Set up a simple pendulum with a leaden ball (or bullet) and a cotton thread (say 50 cms. long). Tie on the bullet another pendulum composed of a rounded cork and a cotton thread. Swing the combined arrangement and study what happens when length of upper pendulum is (1) greater than, (2) equal to, (3) less than, the length of the lower pendulum.

11. Explain the energy changes which occur during the vibrations of a vibrating body. What happens finally to the energy of vibration?

12. Show that the energy of a vibrating body is proportional to the square of the amplitude of vibration. A particle of mass 20 gms. is vibrating 10 times per second with an amplitude of 10 cms. Find (1) its maximum velocity, (2) its energy.

CHAPTER V.

WAVE MOTION.

27. Medium. A medium in the sense used in this chapter is an extended mass of any substance, the outer boundary of which is supposed to be at infinity, or so far removed that it has no influence on the motion of particles in the interior of the mass. Any disturbance, therefore, which may be set up *in* the medium is conditioned only by the properties of the medium (its elasticity, density, and viscosity), and is quite free* from the conditions determined in a more limited mass of the substance by the boundary of the mass.

The atmosphere is a typical example of a free medium. The mass of water some distance below the surface in a lake or in the sea is also a medium in the sense here defined. In fact, the inner portions of any large block of substance may be considered as practically free from boundary conditions, and may act as a free medium.

28. Wave motion involves transmission of vibratory motion through a medium. If a small portion of the substance at any point in a medium be set in vibration, it must necessarily, because of the elastic continuity of the material, communicate its motion of vibration to the adjacent layer. This layer must, for the same reason, communicate the motion to the next layer, and so on from layer to layer. In this way the vibratory motion originated, and maintained, at a certain point in the medium is transmitted outwards from this point through its substance.

* For this reason the term *free medium* is sometimes used.

This transmission of vibratory motion through a medium constitutes *wave motion*.

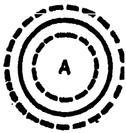


Fig. 19.

Imagine a small sphere A (Fig. 19) in an extended medium to be made to vibrate by expanding and contracting in simple harmonic motion. This vibratory motion is communicated first to the thin spherical shell of the medium immediately surrounding A and is then transmitted, by communication from layer to layer outwards, from A as a centre of vibratory disturbance.

29. The propagation of wave motion occupies time. This process of transmission of vibratory motion from layer to layer of the medium requires more detailed consideration. Let Fig. 20 represent the sphere A surrounded by very thin spherical shells, 1, 2, 3, 4, etc., of the medium; and suppose A to be in its normal position and just beginning to expand. As it expands the layer of particles immediately in contact with it, that is, the inner surface of shell 1, is displaced outwards and a slight compression is set up in the inner layers of the shell. This compression increases and extends

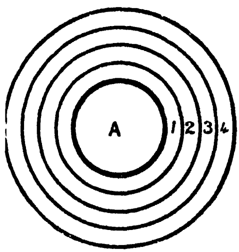


Fig. 20.

into the shell as A continues to expand, and *ultimately, in a very short time after* A *began to expand*, this compression causes the displacement of the inner layer of particles in shell 2, and the disturbance has been transmitted through the first shell of the medium. In the same way the displacement, after reaching the inner layer of particles in shell 2, travels through this shell and in another short interval of time reaches the inner layer of shell 3. In this way the vibratory motion of A is transmitted from layer to layer outwards into the medium. It is important to notice that this process of transmission occupies time; the vibratory motion is communicated

through the shell in a short interval of time; in another similar interval of time. it is transmitted through the second shell, and so on from shell to shell.

If the physical properties of the medium are the same in all directions at any point in it, that is, if the medium is *isotropic*, it is allowable to assume that the motion takes the same time to be transmitted through shells of equal thickness, that is, the vibratory disturbance originated at A may be assumed to be transmitted through the medium with *uniform velocity*.

30. Transmission of displacement. We have seen that the displacements which constitute vibratory motion can, by virtue of the elasticity of the medium, be transmitted through it with uniform velocity.

It remains to consider in detail the state of motion and strain set up in a medium through which vibratory motion is being transmitted.

As A, Fig. 19, vibrates outwards and inwards in simple harmonic motion the spherical layer of particles of the medium in direct contact with its outer surface is thereby set in vibration with the same period, and with practically the same amplitude. This vibratory motion is communicated, step by step, as it is acquired, to the next adjacent layer, but as this process of communication occupies time, the successive steps in the cycle of displacements which constitute the vibratory motion will be communicated to this layer a very short time *after* they occurred in the first layer. That is, the second layer will vibrate in the same way as the first layer, *but it will be later in phase by the short interval of time necessary for the transmission of each stage of displacement from the first layer to the second.* In this way successive layers of the medium will be set in vibration in the same way as A, but the phase will be later and later as the disturbance travels outwards into the medium from A, the difference of phase between any two layers being measured by the time taken by the disturbance in travelling from one layer to the next.

At the end of the first complete vibration the state of the medium surrounding A will be as follows. The

successive stages of displacement which occur in a complete vibration will have travelled outwards from A into the medium for an interval of time extending from the instant at which each occurred until the end of the period. The *initial* stage will therefore have travelled for a complete period into the medium, and will have reached a layer at a distance x from A. The *final* stage, occurring at the end of the period, will not have travelled into the medium at all and will be at A. Between the extreme layer, at a distance x from A, which the initial stage of displacement has just reached, and the layer at A, which is in the final stage of displacement, the successive layers will be found to be in the successive stages of displacement which occur in a complete vibration.

The curve in Fig. 9 has been constructed to represent these successive stages of displacement; it therefore also represents the successive displacements which exist *for an instant* at the end of the first complete vibration of A, in the medium between A and the extreme layer, at a distance x, which the disturbance has just reached.

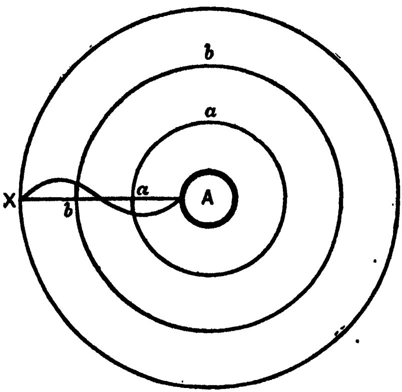


Fig. 21.

Thus, in Fig. 21, if X represent the position of this extreme layer and a curve similar to that of Fig. 9 be drawn on the line X A as axis from X to A, to represent the successive displacements of A

during a complete period, it will also represent the displacements in the successive layers between X and A *at the end* of the first complete vibration of A. In the figure the curve is drawn so that ordinates above the line, X A, represent outward displacements and ordinates below the line inward displacements.* From this curve the phase of vibration of any layer between A and X, *at the instant considered* (the end of the first vibration of A), can be determined. For example, the displacement of *all* the particles in layer *b* is represented by the ordinate at *b*, that is, the layer is expanded outwards by a distance represented by the ordinate at *b* on the curve, and is moving inwards towards its normal position. Similarly the layer at *a* is displaced inwards by a distance represented by the ordinate at *a*, and is moving inwards to its extreme displacement.

The complete motion of any layer during the first vibration of A can also be inferred from the curve.

The layer at *a*, for example, began its vibration when the disturbance reached *a*, that is, A *a*/A X of a period later in phase than the layer at A, and continued its motion while the disturbance travelled from *a* to X. The portion, X *a*, of the curve therefore represents the motion of the layer for this interval of time. Similarly, the layer *b* began its vibration when the disturbance reached *b*, that is, A *b*/A X of a period later in phase than the layer at A (and *a b*/A X of a period later in phase than the layer at *a*), and continued its motion while the disturbance travelled from *b* to X. The portion X *b* of the curve therefore represents the motion of the layer during this interval of time.

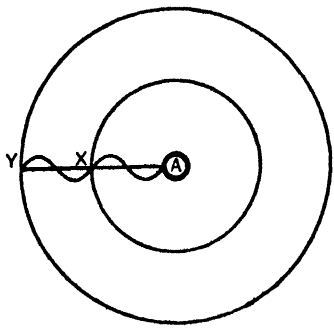


Fig. 22.

Let us now consider how the process of transmission goes on during the second complete vibration of A. From what has been said it will be evident that the disturbance, having travelled out to a distance *x* from A during the first period, will travel out a further distance *x* during the second period, and so reach a layer at Y (Fig. 22), at a distance 2*x*

* It is very important to be clear that the *actual* displacements *are, outwards or inwards, along* A X, but are *represented* in the curve by ordinates at right angles to A X.

from A. Also, at the end of the second period the layers of the medium between X and Y will evidently be in the same phases of vibration as the corresponding layers between A and X were at the end of the first vibration. Further, the layers of the medium between A and X will each have performed a *complete* vibration during this second period, and will therefore be again in exactly the same phases at the end of the second period as they were at the end of the first period. The medium between X and Y is therefore in exactly the same state as that between A and X, the only difference being that the layers between A and X have each performed one complete vibration more than the corresponding layers between X and Y.

If a curve similar to that drawn in Fig. 21 for one period from X to A be drawn for two periods from Y to X and from X to A, as in Fig. 22, it will evidently represent the displacements of the successive layers from A to Y *at the end* of the second complete vibration of A.

It will now be clear that during the third and succeeding periods of the vibration of A, the disturbance, which has its origin at A, travels a further distance x into the medium during each succeeding period, and that, *at the end of each period*, the state of the medium in the successive shells of thickness x, such as A to X, X to Y, and so on, will be exactly the same for each shell and similar to that described in detail for the shell AX at the end of the first period. The state of the medium in each shell will be the same, not only at the end of each period, but at any instant during any period. For, if at the end of a period the corresponding layers of two adjacent shells are in the same phase, they must always be in the same phase, for they are vibrating with exactly the same period.

31. Development and transmission of strain in the medium. We have, so far, considered in some detail the transmission of the vibratory motion from A into the surrounding medium by considering the propagation of *displacement* through the medium. It will be evident, however, that this transmission of vibratory motion involves not only the transmission of periodic displacement from

layer to layer, but also the development and transmission of periodic *strain* from layer to layer.

Following the line of argument adopted in Art. 21, it is evident that, for a thin shell of air surrounding A, unless the inner and outer surfaces suffer, at the same instant, equal displacements, the shell must suffer either compression or rarefaction according as its surfaces are brought nearer together or drawn further apart as the result of any difference in their respective displacements. The strains developed and propagated in the medium are therefore strains of *compression* and *rarefaction*.

The curve X A, in Fig. 21, represents the displacements which exist at the end of the first period in the layers of the medium between A and X. At the end of the second period these displacements have travelled out into the shell X Y (Fig. 22) and they again recur in the shell A X. Thus the cycle of displacements which each layer undergoes during a complete vibration and which is transmitted step by step, from layer to layer, is completely represented by the curve X A. Hence, if we can determine the strains which accompany the displacements represented by this curve, we can specify completely the sequence of strains which exist between Y and X and between X and A at the end of a complete vibration of A, and also the cycle of strains which each layer undergoes during a complete vibration and which is transmitted, stage by stage, from layer to layer.

The strains which accompany the displacements represented by the curve X A are readily determined. The curve, here reproduced (Fig. 23), is drawn so that outward displacements of a layer are represented by ordinates drawn above the line X A, and inward displacements by ordinates drawn below the line. Now, for any two layers close together, say at P and Q, the strain in the medium between them is determined by the difference of the ordinates at P and Q. Both layers are here displaced outwards, but the layer at Q is displaced further than the layer at P. The medium between the layers is therefore *compressed*, and the degree of compression or *strain* is measured, as explained in Art. 21, by the ratio $pq\,/\,PQ$. From a geometrical point of view this ratio, $pq\,/\,PQ$, determines the slope of the curve between P and Q. Hence we get the result that the slope of the displacement curve between any two points a very short distance apart, as measured by the ratio of the difference of the ordinates at these points to the distance between the points, measures the strain in the medium between

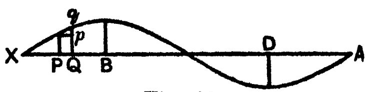


Fig. 23.

these points. It follows that so long as the slope of the curve remains the same in direction, the strain is of the same character (compression or rarefaction), and that when the slope changes direction the strain also changes in character. Thus, in Fig. 23, the curve slopes upwards from X to B and from D to A, and downwards from B to D; the strain in the medium is therefore of the same character from X to B and from D to A, and of the reverse character from B to D. As the curve is drawn it has been shown that the medium between P and Q is in compression, therefore we have compression from X to B and from D to A, and rarefaction from B to D.

In this way we can determine the strain at any point in the medium between A and X at the end of the first or any succeeding period of the vibration of A. For, if we take the curve of displacement XA (Fig. 24) and divide the distance XA into a number of very small equal distances determined by the points X, a, b, c, . . . q, A, then the strain in the medium between any two adjacent points is measured by the ratio of d, the difference of the ordinates at these points, to the distance D between the points.

Fig. 24.

We may therefore obtain a curve showing the state of strain in the medium between X and A, by determining graphically from the displacement curve the strain in the medium for the successive thicknesses, Xa, $a b$, $b c$, . . . q A, and plotting the value obtained for each thickness as the mean value of the strain in that thickness at its middle point. If we represent compressions by ordinates above the line and rarefactions by ordinates below the line the strain curve obtained in this way for the medium between X and A at the end of the first or any succeeding period is of the form shown in Fig. 25.

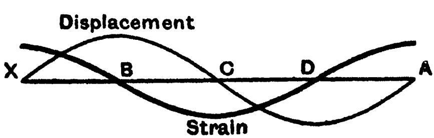


Fig. 25.

It will be seen that from X to B the strain is one of gradually decreasing compression; from B to C gradually increasing rarefaction; from C to D gradually decreasing rarefaction; and from D to A again gradually increasing compression. It can be shown that if the displacement from X to A varies in the same way as the sine of an angle from 0° to 360°, the strain varies in the same way

as the cosine of an angle from 0° to 360°. The curves also show that the strain is a maximum where the displacement is zero, and that as the displacement increases the strain decreases, until, as the displacement reaches its maximum value, the strain is zero. It is useful to note that the strain in the medium at any point is proportional to the velocity of the vibrating particles at that point (Art. 3), and that when the particles are moving in the direction of transmission the state of strain is one of compression, and when they are moving in a direction opposite to that of transmission the strain is one of rarefaction.

The strain curve obtained in this way represents the strain in the medium between X and A *for an instant* at the end of a complete vibration. Just, however, as the displacement curve represents the change in the displacement of a particular layer or particle during a complete vibration, so the strain curve here obtained may represent the change of strain or the cycle of strains which occurs in a particular layer during a complete vibration. That is, the strain and displacement curves may represent either the states of strain and displacement which occur in the medium *between X and A at a particular instant*, or the cycle of strains and displacements which occur at *a particular point in the medium during a complete vibration.*

EXERCISES V.

1. Explain what is meant by *longitudinal wave motion* in a medium. Describe the process of propagation of longitudinal wave motion in a free medium.

2. What do you mean by the *compressions* and *rarefactions* of a longitudinal wave system?

3. What arguments would you bring forward to prove that longitudinal waves travel with uniform velocity through an isotropic free medium.

4. Explain how to get a strain diagram from a displacement diagram of longitudinal wave motion. If the displacement diagram is a sine curve show that the strain diagram is also a sine curve.

5. Show that a displacement curve showing the displacements of all points in a medium between any two points A and B of equal phases in the line of a longitudinal wave, also shows the cycle of displacements which occur at any particular point.

6. Show that the strain at a point in a medium through which longitudinal waves are passing is proportional to the velocity of that point, also that a particle which is in compression is moving away from the source while a particle which is in rarefaction is moving towards the source.

CHAPTER VI.

WAVE MOTION—continued.

32. General character of wave motion. From what has been said in the preceding chapter it will be clear that the vibrating body A (Fig. 19) acts as a centre of disturbance in the medium surrounding it. The vibration of A is transmitted outwards in all directions into the medium with a definite uniform velocity, so that at the end of n periods the disturbance has penetrated to a distance nx into the medium. Each layer of the medium, as the disturbance reaches it, begins to vibrate outwards and inwards about its normal position in the same way as A, but as it begins to vibrate later than A by the time taken by the disturbance in reaching it, it is always later in phase than A by this interval of time. This transmission of vibratory motion involves not only the transmission through each layer of the sequence of displacements which constitute the vibratory motion, but also the transmission of the sequence of strains which are set up at each point as the result of a difference in the displacement of adjacent layers.

It is important to notice that in the propagation of wave motion no portion of the medium is permanently displaced from its normal position. Each layer, as the wave is transmitted through it, is merely set in vibration about its normal position, and this vibration is transmitted from layer to layer with gradual retardation of phase as the wave travels on.

33. Crova's disc. The distribution of displacement and strain in a medium through which longitudinal wave motion is passing is represented diagrammatically in Fig. 26. Each layer here represented is displaced as indicated

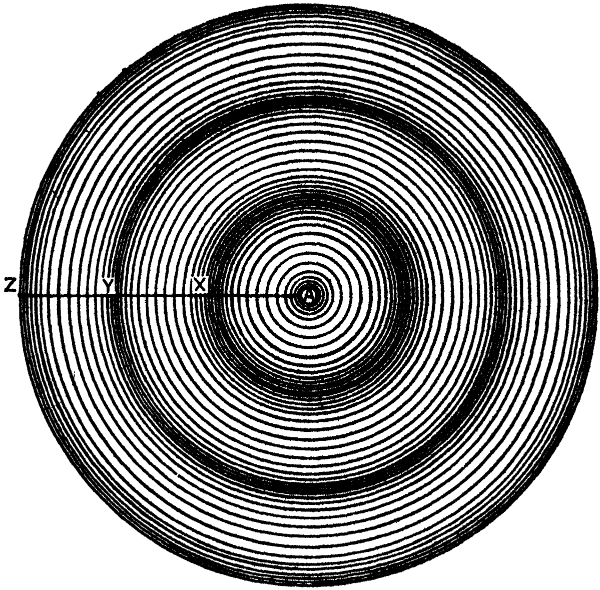


Fig. 26.

by the displacement curve of Fig. 9, and the resulting compressions and rarefactions are clearly shown in the figure.

The actual process of the propagation of wave motion, with its attendant displacements and strain, may, however, be shown very clearly by an ingenious arrangement known as *Crova's disc* (Fig. 27). This disc is constructed as follows. Describe a small circle of, say, 2 mm. radius at the centre of the disc. Take 8, 10, 12, or more equidistant points on the circumference of this circle, and

with these points, taken in order round the circle for as many revolutions as we please, as centres, describe a series of circles with radii gradually increasing by a constant amount, say 2 mm., not less than the distance between the points on the circumference of the small central circle. If the disc be now made to rotate round an axis through the centre of the small circle, it is evident from the construc-

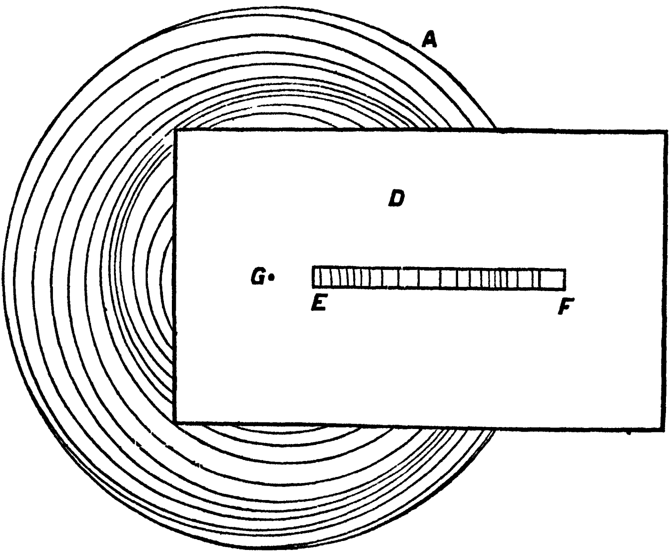


Fig. 27.

tion that the circumference of each circle, at the point where it passes any *fixed* radial line, such as the edge of a card held radially in front of the revolving disc, moves backwards and forwards along the line in (approximately) simple harmonic motion of amplitude equal to the radius of the small central circle. Further, the phase of vibration for each circle gets later and later as we pass outwards from the centre by $1/n$ of a period for each circle, if n denote the number of equal parts into which the small

central circle has been divided. Hence if a card D with a rectangular slit E F, arranged so as to show short arcs of the circumferences of the circles along any radial line, be set up in front of the revolving disc, the motion of the row of arcs, seen through the slit as the disc revolves, represents the transmission of vibratory motion from arc to arc, and the states of compression and rarefaction which accompany this transmission are seen to form and travel from point to point along the row.

34. The wave machine. The propagation of longitudinal wave motion is conveniently illustrated by an arrangement which is sometimes called the wave machine. It consists

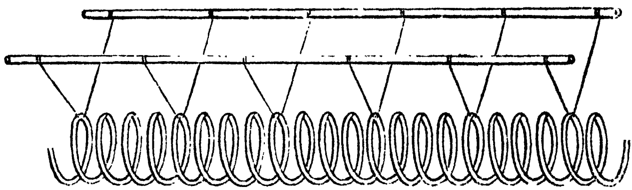


Fig. 28.

of a helix of stout brass wire, about six feet long and three inches in diameter, suspended from two parallel bars by threads looped round several turns of the helix as shown in Fig. 28.

If the end of the helix be sharply pushed in, the nearest turns are compressed, and the compression can be seen travelling along the coil, each turn of the helix moving *forward* a little as the compression reaches it. Similarly if the end of the helix be pulled sharply outwards the end turns are slightly separated or extended, and the extension may be seen travelling along the coil, each turn of the helix moving *backward* a little as the extension reaches it. Hence it follows that if one end of a very long helix be set in periodic vibration, longitudinal wave motion of compression and extension may, with proper precautions, be propagated along the helix.

35. Form of wave front. In the case dealt with in the preceding articles A is the centre of the vibratory disturbance, and the wave of displacement and strain travels outwards from A into the medium with a definite uniform velocity determined by the elasticity and density of the medium. The source of disturbance A is here supposed to vibrate by periodic expansion and contraction, always remaining spherical in form. It therefore sends out from the beginning of its motion a symmetrical disturbance such that the front of the wave advancing into the medium is always spherical. In this case, therefore, the *wave front* at any instant, or the locus of all points in the medium which the disturbance has just reached at that instant, is spherical from the beginning of the motion.

It will be understood, however, that *any* vibrating body may act, like A, as a centre of disturbance and originate wave motion in the surrounding medium. In general the form of the wave front for any vibrating body will at first be determined largely by the shape of the body, but as the disturbance travels out the front tends more and more to the spherical form, so that at a distance from the vibrating body, very large compared with the dimensions of the body, the form of the wave front is practically spherical.

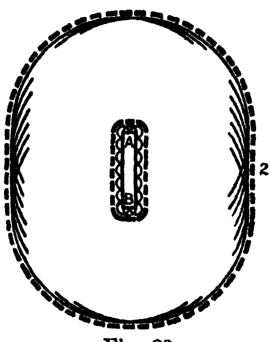


Fig. 29.

For example, suppose A B (Fig. 29) to represent a vibrating body such as a rod in longitudinal or transverse vibration in air as the surrounding medium. Then in a very short interval of time the disturbance from each *point* of the vibrating body will have reached all points in the medium on the surface of a small sphere having the point of disturbance for centre, and of radius equal to the distance travelled by the disturbance in the short interval of time considered. If then these small spheres are supposed to be drawn round all points on the surface of A B, as indicated in section for a few points in the plane of the paper in Fig. 29, the outer surface which is tangential to or envelops these small spherical

surfaces externally is evidently the *wave front* of the disturbance at the end of the small interval of time for which the spheres have been drawn. The trace of this wave front is indicated by the dotted line *1* in the figure. Its form is evidently very similar to that of the vibrating body A B.

If the wave front at the end of a longer time be determined in exactly the same way, its form as indicated by the trace represented by the dotted line *2* in the figure, while still somewhat flattened at the sides, evidently tends to become spherical. At a distance from A, much greater than the dimensions of A, the divergence of the form of the wave front from a spherical surface will be negligibly small.

36. Wave length. In Art. **30** it has been explained that during the time of the first complete vibration of A the vibratory disturbance travels a distance x into the medium, and during the next vibration it travels on a further distance x, and so on, vibration after vibration. This distance, through which the wave motion travels in the medium in the time of one complete vibration of the source of disturbance, is called the *wave length* of the motion in that medium.

It has also been explained that in the medium round A the difference in phase of vibration between any two layers a distance x apart is one complete period, that is, any two layers a distance x apart are in the *same phase* of vibration. The wave length of the motion may therefore also be defined as the shortest* distance between two layers in the same phase of vibration.

The wave length of the disturbance originated by the same vibrating body will evidently be different in different media. For in different media the velocity of transmission will be different, and the distances travelled by the disturbance during the period of vibration will therefore also be different for the different media. For example, if a body making 100 vibrations per second vibrate in air the wave length of the wave motion in air will be the distance the wave motion travels *in air* in 1/100 second; but if the body vibrate in water then the wave length will be the distance the wave

* Layers at distances $2x$, $3x$, $4x$, etc., apart are also in the same phase of vibration.

motion travels *in water* in 1/100 second. As the velocity of the wave motion will not be the same in media with such different physical properties as air and water the wave lengths of the wave motion in these media must be different.

From what has been said it will be plain that if λ denote the wave length of a disturbance from a source of frequency n in a medium for which the velocity of transmission of the wave motion is V, then $V = n\lambda$. For, by definition, λ is the distance which the wave motion travels during one period of vibration, and $n\lambda$ is therefore the distance it travels during n periods, or in one second. But the distance the motion travels in one second is its velocity, therefore we have $V = n\lambda$.

It should be carefully noticed that in this relation V and λ must always refer to the same medium. In medium A we have $V_A = n\lambda_A$, and in medium B, $V_B = n\lambda_B$, where V_A and V_B denote the velocity of the motion in media A and B respectively, and λ_A and λ_B the corresponding wave lengths in these media.

37. Longitudinal wave motion. The wave motion dealt with in preceding articles starts out from the vibrating body A and travels outwards *radially* from A as a centre of disturbance. The direction of vibration of the layers of the medium through which the motion passes is also *radial*, that is, the vibration of any spherical layer of particles is outwards and inwards *radially*, each particle vibrating to and fro *along the radius* passing through it. Hence in this case of wave motion the path of vibration for each particle to which the motion is transmitted is along the line of transmission. Thus for the row of particles along A X the wave motion travels from A to X along this line and the vibration of each particle is backwards and forwards, with a small amplitude on each side of its normal position, along this line.

Wave motion of this kind, in which the vibratory motion transmitted along any line of particles is along the line of transmission, is called *longitudinal wave motion*.

Longitudinal wave motion, as has been explained in detail, is always accompanied by the transmission of periodic compression and rarefaction through the medium. A longitudinal wave in any medium may therefore be spoken of as a wave of compression and rarefaction.

38. Transverse wave motion. In any case in which the vibratory motion transmitted along a line of particles is at right angles to the line of transmission, or in a plane at right angles to this line,* the resulting motion constitutes what is known as *transverse wave motion.*

Let A (Fig. 30) represent a thin plane layer, taken at right angles to the direction X Y, in the mass of an extended medium.

If A be displaced slightly in any direction in its own plane it will tend to displace similarly adjacent layers parallel to it only if the medium is one which possesses the property of *simple rigidity*. Of material media only a solid possesses simple rigidity, hence in solid media a displacement of A in its own plane, at right angles to Y X, will be transmitted to adjacent parallel layers on both sides of A in the direction A X and A Y. In a liquid or gaseous medium the displacement of the layer A in its own plane would produce a very slight displacement of adjacent parallel layers as the result of the slight molecular friction between adjacent layers, which constitutes the viscosity of an imperfect fluid. This slight displacement due to viscosity will not be transmitted to any great distance from A, on account of the rapid dissipation of energy which occurs when work is done against frictional resistance.

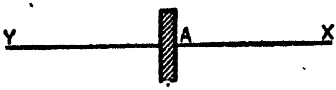


Fig. 30.

As the displacement of A in its own plane in a *solid* medium can be transmitted to adjacent parallel layers, it follows that if A is set in vibration in its own plane, at right angles to Y X, the motion of vibration will be transmitted from layer to layer in the directions A X and A Y. That is, this vibratory motion of A will originate transverse wave motion in the medium in the directions A X and A Y. If, now, we can imagine a source of disturbance at a point A in a solid medium capable of originating transverse wave motion in *all* directions radiating from A, then A would become a centre of disturbance from which transverse wave motion with a spherical

* The path of vibration might be a circle, or ellipse, or other curve in a plane at right angles to the line of transmission.

wave front is propagated outwards into the medium in the same way as, in the case represented by Fig. 22, A is a centre of disturbance from which longitudinal wave motion with a spherical wave front is propagated out into the surrounding medium.

From what has been said it will be clear that longitudinal wave motion may be originated in any medium, solid, liquid, or gaseous, which possesses elasticity of bulk, but that transverse wave motion can be originated only in a medium, such as a solid, which possesses elasticity of form or simple rigidity.

Ether, the medium which is supposed to pervade all space and all matter, is capable of transmitting transverse wave motion, for, as has been explained elsewhere, *radiation* is transverse wave motion in the ether. Similarly it may be said that *sound* is longitudinal wave motion in any material medium. In this section of the book we have therefore to do mainly with longitudinal wave motion in material media.

39. Transmission of transverse vibratory motion along a line of particles in a medium. Let the dots along the line A B (Fig. 31) represent a few of the particles in the

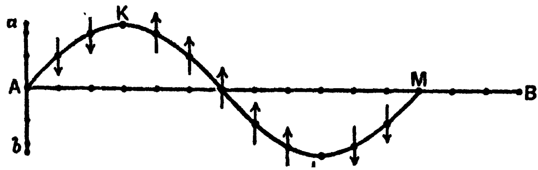


Fig. 31.

medium along the line A B, and imagine the transverse vibratory motion of the particles to originate at A. The particle at A may be supposed to vibrate up and down along the path *a b* taken at right angles to A B, and this vibratory motion is, by virtue of the elasticity of the medium, transmitted from particle to particle along the line A B with a retardation of phase determined by the

time the disturbance takes to travel from particle to particle.

At the end of a complete vibration of the particle at A over the path A b a A it is evident that the disturbance will have travelled out along A B to a point, represented by M in the figure, and that, as a result of the retardation of phase, the particles between A and M will, at that instant, be in positions such that they are arranged along the curve A K L M, each particle moving along its individual transverse path in the direction indicated by the arrow in the figure. At the particular instant considered the particles between A and K and between L and M are evidently all moving downwards, and those between K and L are moving upwards; the strain at any point in the medium is the strain associated with the difference of the *transverse* displacements of two adjacent particles. The wave length of the disturbance is represented in the figure by the distance A M, and, as the figure is drawn, the particles represented along A K L M differ in phase by one-twelfth of a period.

The curve A K L M, which is here seen to represent the position of the particles during the transmission of a particular case of transverse wave motion along a line of particles in a medium, is often used as a conventional representation of wave motion of any kind along a line.

40. Transmission of transverse displacement along a stretched flexible string. If any point in a stretched flexible string is displaced transversely the displacement will be transmitted, by virtue of the tension in the string, from the point of displacement along the length of the string with a definite velocity.

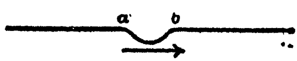


Fig. 32.

Hence, if a small portion of the string be deformed by lateral displacement, as shown at $a\,b$ (Fig. 32), under conditions such that it travels along the string in the direction indicated by the arrow, each of the displacements which determine the deformation travels with the same velocity, and the deformation therefore travels as a whole

along the string without change of form. This deformation, or *pulse*, of lateral displacement travels, therefore, along the string without change of form and at a definite speed in such a way that every portion of the string suffers the deformation as the pulse passes over it, and returns to rest in its initial straight position as soon as the pulse has passed.

In general, if a deformation is produced at any point in a stretched string, it gives rise to two similar component pulses, which travel in opposite directions along the string from the point of deformation.

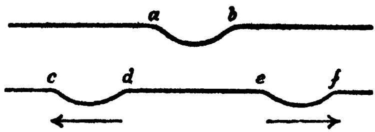


Fig. 33.

As shown in Fig. 33, the amplitudes of these component pulses, cd and ef, are half the amplitudes of the initial deformation ab, so that if we imagine them to be recombined they would give the original deformation as their resultant.

If the string is stretched between fixed points a pulse of transverse displacement when it reaches a fixed point is reflected, as

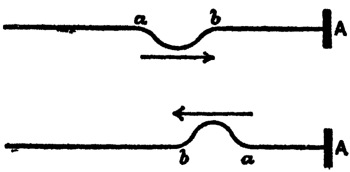


Fig. 34.

explained later, with reversal of displacement, as shown in Fig. 34, where the pulse ab is shown before and after reflexion at the fixed point A.

If transverse periodic motion is set up and maintained at any point in a stretched string of unlimited length, it causes *waves* of transverse displacement to travel along the string, but, for certain reasons, the amplitude of the waves decreases rapidly as they travel away from their source.

If a stretched string or wire is plucked or otherwise displaced at any point, the pulse of displacement is in general of such small amplitude, and travels so quickly that, though it may be felt passing along the string, it cannot easily be seen. If, however, a long thick indiarubber tube or a stout flexible rope be suspended from the ceiling and weighted at the lower end, the pulse produced when it is plucked aside at the lower end can be seen to travel up the rope and to be reflected at the top end with reversal of displacements.

41. Ripples and waves on the surface of a liquid. Ripples and waves on the surface of a liquid are familiar examples of wave motion. In the case of ripples only the particles near the surface layer of the liquid are involved in the motion; these particles vibrate up and down in vertical paths, which are approximately straight lines at right angles to the direction of transmission, and the motion is essentially transverse wave motion along the surface sheet of the liquid. In the case of larger waves

Fig. 35.

particles below the surface are involved to a depth determined by the size of the waves. These particles vibrate, as shown in Fig. 35, in closed curves lying in a vertical plane through the direction of transmission at any point. The wave motion in the case of these waves is therefore not strictly transverse wave motion, but under the conditions of transmission the transverse component of the vibratory motion of the particles is the one that determines the character of the waves, and the wave motion may therefore be considered as a case of transverse wave motion.

The motion of the particles in water waves may be observed on a small scale in water in a trough such as is shown in Fig. 35. A wave may be started by dipping the end of a block of wood into the water at one end of the trough, and if the water contains a number (not too many) of particles of paper pulp, the character of the wave motion may be studied by observing the motion of these particles.

If a piece of cork is placed on the surface of water, along which regular surface waves are being transmitted, it will be noticed that it is not carried along by the waves, but oscillates up and down at a particular point in a closed vertical curve with a definite period. This period is the period of the wave motion, and may be determined by direct observation of the motion of the piece of cork.

As a result of the transmission of the vibratory motion of the surface particles from point to point along the surface, the particles along any line of transmission arrange themselves in a wave curve similar to that indicated, for a wave length, by A K L M in Fig. 31, and the surface of the liquid thus assumes the well-known *trough* and *crest* wave form characteristic of ripples and waves on a liquid surface. At any point where a crest occurs the particles are evidently at the highest point of their path of vibration, and at any point where a trough occurs the particles are evidently at the lowest point of their path of vibration. Hence, since the particles are in the same phase at the crests and also at the troughs, it follows that the wave length may be determined by measuring the distance from crest to adjacent crest or from trough to adjacent trough.

Thus, in the case of these waves, the period and wave length can be readily determined by simple observations, and the velocity of transmission of the waves can therefore be easily found. For, if n denote the frequency (which is found as the reciprocal of the period) and λ the wave length, then, as explained in Art. **36**, V, the velocity of transmission, is given by the relation $V = n\lambda$.

It can be shown that in the case of ripples, or waves whose wave length does not exceed a certain limit, the

force involved in the transmission of the waves is mainly the surface tension of the liquid. When the wave length exceeds this limit the waves become *gravitational waves*, and the force involved in their transmission is mainly the force of gravity. In the case of water the limiting wave length for ripples is about 1·5 cms. For wave lengths greater than this, water waves become gravitational in character, and the familiar sea and ocean waves are examples of gravitational waves on a grand scale.

EXERCISES VI.

1. Describe the construction of Crova's disc and show that it very closely exhibits longitudinal wave motion.

2. The wave front due to a body of irregular shape vibrating in a free medium is irregular in shape near the body but is spherical far away. Explain this.

3. Define *wave length*, and establish the relation between the wave length and the velocity of wave motion in a free medium.

4. A body vibrating with frequency 100 sends waves 10 cms. long through a given medium. Find the wave velocity in this medium.

5. A body vibrating with a constant frequency sends waves 10 cms. long through a medium A and 15 cms. long through a medium B. The velocity of the waves in A is 90 cms. per second. Find the wave velocity in B.

6. Longitudinal waves 10 cms. long travel through a medium with a velocity of 1100 cms. per second. Find the frequency of vibration of the vibrating body.

7. What is meant by *transverse wave motion?* Give examples of this type of wave motion.

8. Why cannot transverse waves be transmitted through liquids and gases?

9. Explain the difference in the formation of large water waves and ripples. Show that the former waves are not true transverse waves except on the surface itself.

CHAPTER VII.

VELOCITY OF PROPAGATION OF LONGITUDINAL WAVE MOTION IN A FLUID MEDIUM.

42. Velocity of propagation of longitudinal wave motion in a fluid medium. The velocity of propagation of longitudinal wave motion in a fluid medium which possesses no rigidity or elasticity of form is determined by the bulk elasticity and the density of the medium. If E denote the modulus of volume, or bulk elasticity, and D the density of the medium, then V, the velocity of longitudinal wave motion in the medium, is given by the relation

$$V = \sqrt{E/D}.$$

This relation may be established by the following elementary method which, although it does not deal with all the difficulties of the subject, is sufficient to give a general idea of the principles involved in the proof.

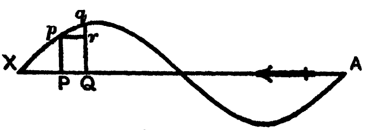


Fig. 36.

Relation between wave velocity and particle velocity for longitudinal wave motion in a fluid medium. Let the curve X A, Fig. 36, represent the curve of displacement for wave motion travelling from A to X, outwards into the medium. The velocity with which the motion is propagated along A X is evidently the velocity with which any particular displacement travels along A X. Let Q and P

be two points taken very close together on the line A X. When the displacement, P p, at P has increased until it is equal to the displacement, Q q, at Q, the displacement at Q is said to have "travelled" from Q to P. This implies that the displacement at Q "travels" from Q to P in the *same time* as the particle at P increases its displacement (along A X) from a distance represented by P p to one represented by Q q, that is, by a distance $r\,q$, the difference between Q q and P p. Hence, if V denote the velocity of the wave motion, t the time in which any given displacement travels from Q to P, and v the average velocity of the particle at P during this time, we have

$$V\,t = QP \text{ and } v\,t = r\,q.$$

Hence we get the relation

$$v/V = r\,q/QP \quad \text{.............................(1)}$$

If the point Q is supposed to be very near, infinitely near, P, then v denotes the velocity of the particle at P in the path of its vibration, and $r\,q/QP$ or $r\,q/r\,p$ denotes, as explained in Art. 31, the *strain* in the medium between P and Q, that is at P, since the distance P Q is infinitely small. In the case of longitudinal wave motion the elasticity of the medium which is involved is volume or bulk elasticity. Hence, if E denote the modulus of volume elasticity for the medium, then, as explained in Chapter II., the relation between stress and strain at any point in the medium is given by the relation

$$\frac{\text{stress}}{\text{strain}} = E.$$

The stress at any point would be measured by the *excess* of the pressure at that point over the normal pressure. Hence, if p denote this excess of pressure at P, we get

$$\frac{p}{r\,q/r\,p} = E \text{ or } r\,q/r\,p = p/E.$$

Substituting this in relation (1) above we have

$$v/V = p/E \quad \text{.................................(2)}$$

This is an important general relation between the quantities involved at any point on the line of transmission.

Dynamics of the motion. If we now consider the motion of any very thin plane layer of the medium taken at right angles to the line of transmission A X, it will be evident that the force *per unit area* causing the motion of the layer backwards or forwards along A X at any instant is the difference of the pressures at the two faces of the layer.

Hence, if we take the layer between the points P, Q on A X (Fig. 37), and if p_1 and p_2 denote the excess of pressure in the medium at the points Q and P respectively at a given instant, then $p_1 - p_2$ is the force per unit area causing the motion of the layer. If x denote the thickness of the layer and D the density of the medium, then the *mass* acted on by the force $p_1 - p_2$ is evidently D x, and, applying the usual relation between force, mass, and acceleration, we have

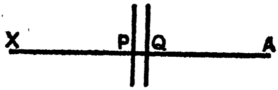


Fig. 37.

$$\frac{p_1 - p_2}{\mathrm{D}x} = a,$$

where a denotes the acceleration of the layer. If Q is supposed to be infinitely close to P a will denote the acceleration at P. But if v_1 denote the velocity of a particle at Q and v_2 the velocity of a particle at P at the given instant, then v_2 will change to v_1 in the time the motion takes to travel from Q to P. This time being x/V we at once get

$$\frac{v_1 - v_2}{x/\mathrm{V}} = a,$$

where a again denotes the acceleration at B if the distance B C is infinitely small.

Equating these two values of the acceleration at B we get

$$\frac{p_1 - p_2}{\mathrm{D}x} = \frac{v_1 - v_2}{x/\mathrm{V}}$$

or
$$\frac{p_1 - p_2}{v_1 - v_2} = \mathrm{V\,D}.$$

But, from the relation expressed by (2) above,

$$\frac{p_1 - p_2}{\mathrm{E}} = \frac{v_1 - v_2}{\mathrm{V}}$$

or
$$\frac{p_1 - p_2}{v_1 - v_2} = \frac{\mathrm{E}}{\mathrm{V}}.$$

Hence we get

$$\mathrm{V\,D} = \frac{\mathrm{E}}{\mathrm{V}} \text{ or } \mathrm{V}^2 = \mathrm{E}/\mathrm{D}.$$

That is, $$\mathrm{V} = \sqrt{\mathrm{E}/\mathrm{D}}.$$

43. Application of the formula in special cases. The velocity of longitudinal wave motion in a fluid medium is therefore given by the relation

$$V = \sqrt{E/D},$$

where E denotes the modulus of volume elasticity and D the density of the medium. In applying this relation it must be remembered that E is the modulus of elasticity which properly applies to the strain set up in the medium. In a free unlimited fluid medium, where the strain is a pure volume strain, the modulus applicable is the modulus of volume elasticity. If the conditions of the strain are isothermal or adiabatic, then the modulus of isothermal or adiabatic elasticity must be taken. Since the strains of compression and rarefaction which accompany longitudinal wave motion are very suddenly produced and of very short duration, they must, in a medium of low thermal conductivity, take place under practically adiabatic conditions, and the modulus of elasticity involved is therefore the modulus of adiabatic elasticity of volume. Hence, in the case of a gas, the value of E is γ P, as explained in Art. 12, and the velocity of longitudinal wave motion in a gas is therefore given by

$$V = \sqrt{\gamma P/D},$$

where P denotes the pressure of the gas, γ the ratio of its two specific heats, and D its density.

In a solid medium E is replaced by a quantity which involves the modulus of volume elasticity and the modulus of simple rigidity or form elasticity.

In the case of the transmission of longitudinal wave motion along the length of a thin solid rod the conditions of strain are those to which Young's modulus applies; this modulus must therefore be taken to determine the velocity in this case.

The density of the medium, represented by D, is the density under the conditions that exist at the time of propagation of the wave motion.

EXERCISES VII.

1. When a longitudinal wave is travelling through a medium show that at any point

$$\frac{\text{the particle velocity}}{\text{the wave velocity}} = \frac{\text{the excess pressure}}{\text{the elasticity}}.$$

2. Prove that the velocity of longitudinal waves through a medium

$$= \sqrt{\frac{\text{elasticity}}{\text{density}}}.$$

Which elasticity must be taken?

3. What does the above formula become in the case of a gas?

4. Young's modulus for steel = 2×10^{12} dynes per sq. cm. The density of steel is 7·7 gms. per c.c. Find the velocity of longitudinal waves through steel.

5. The velocity of longitudinal waves through water = 143500 cms. per second. Find the elasticity.

6. The relation between the pressure and volume of a gas kept at constant temperature is PV = a constant.

The relation between the pressure and volume of a gas when no heat is allowed to escape is PV^{γ} = a constant.

Show from the isothermal relation that the isothermal elasticity is equal to the pressure, and from the adiabatic relation that the adiabatic elasticity is equal to γ times the pressure.

7. Why in the formula $V = \sqrt{\frac{E}{D}}$ must the adiabatic elasticity be used and not the isothermal elasticity?

8. The velocity of longitudinal waves in hydrogen at 0° C. = 1280 metres per second. Find the velocity in oxygen at 0° C. (oxygen is 16 times as dense as hydrogen).

CHAPTER VIII.

ENERGY OF WAVE MOTION.

44. Energy of wave motion. A medium through which wave motion is passing possesses energy in virtue of this motion. This energy is partly kinetic and partly potential; the particles of the medium in vibratory motion possess kinetic energy and the portions of the medium under strain possess potential energy of strain. This energy is communicated to the medium by the vibrating body at which the wave motion has its origin. The energy communicated to the medium in successive complete vibrations travels out with the motion, and is found in successive shells of the medium of thickness equal to the wave length of the disturbance. Thus, in the case of Fig. 22, a quantity of energy equivalent to that communicated by A to the surrounding medium in its first, second, and third complete vibrations is located in the outer, middle, and inner shells of thickness x between Z and Y, Y and Z, and X and A respectively.

If q denote the quantity of energy communicated by the vibrating body to the medium in one complete vibration, then q also denotes the energy distributed throughout any shell of wave length thickness in the medium. It can be shown that at any instant half of this energy is kinetic energy and half potential energy of strain.

45. Intensity of wave motion at any distance from the source: law of inverse squares. The energy distributed in any shell of wave length thickness travels outwards through the outer surface of the shell in the period of vibration of the source. If, therefore, n be the frequency of the source and q the energy communicated to the

medium in one period, the quantity of energy transmitted across any spherical surface in one second is constant and equal to $n q$. This implies that the quantity of energy transmitted in one second across *unit area* of any spherical surface, having the source of disturbance as centre, decreases as the radius of the surface increases. The total quantity of energy transmitted through the whole surface in one second is constant, but the area of the surface increases directly as the square of its radius: the quantity of energy transmitted across unit area per second must therefore vary inversely as the square of the radius.

The *intensity* of the wave motion disturbance which reaches any point from a centre of disturbance is usually measured by the quantity of energy transmitted per second across unit area of the spherical wave surface through that point. The intensity of the disturbance transmitted to any point from a centre of disturbance must therefore vary inversely as the square of the distance of the point from the centre.

This law of inverse squares as here stated applies strictly for all distances only when the source of disturbance is a vibrating particle or sphere. In this case the wave front is always spherical; the energy transmitted per second across unit area of any spherical surface round the source as centre is therefore the same at all points on the surface, and the intensity of disturbance at any point varies inversely as the square of the distance of the point from the centre of disturbance. When the source of disturbance is a vibrating body of some size, the wave fronts near the source, as explained in Art. 35, are not spherical, and the quantity of energy transmitted per second across unit area of a surface having the form and position of these wave fronts varies from point to point on the surface. The intensity at any point near the source therefore depends upon its position relative to the source as well as upon its distance from it.

For points at a great distance from the vibrating body the law of inverse squares may, however, be applied. When the distance is great compared with the dimensions of the source of disturbance, the wave front is practically

spherical, and the source may be considered as a *point source* at the centre of the wave front.

46. Law of variation of amplitude of vibration with distance from the centre of disturbance. As the intensity of the disturbance from any vibratory source varies inversely as the *square of the distance* from the source, it follows that the amplitude of vibration of the particles of the medium must vary inversely as the *distance* from the source. The intensity of the disturbance at any point is directly proportional to the *square* of the amplitude of vibration at that point; hence, if the intensity varies inversely as the square of the distance from the source, the amplitude must vary inversely as the first power of the distance. Thus, for two points at distances in the ratio 1 to 2 the intensities will be in the ratio 4 to 1, and the amplitudes of vibration in the ratio 2 to 1. This is evidently in accord with the law that the intensities at the two points should be in the ratio of the *squares* of the amplitudes at the points.

It can in fact be shown that the intensity of a wave disturbance from a vibrating source at any point is given by $2\pi^2 n^3 r^2 \lambda d$, where n denotes the frequency of the source, r the amplitude of vibration of the medium particles at the given point, λ the wave length, and d the density of the medium. For, if we imagine a spherical wave front at a distance R from the source to advance through a *very small* distance x into the medium, then energy has been communicated to a shell of the medium of volume $4\pi R^2 x$ in a time x/v, where V is the velocity of the disturbance in the medium. If N denote the number of particles or molecules in unit volume of the medium and q the energy communicated to each particle, then the energy communicated to the volume $4\pi R^2 x$ *per second* is $4\pi R^2 x N q V/x$ or $4\pi R^2 N q V$ across the surface of area $4\pi R^2$, and the energy transmitted per second across unit area of this surface is therefore given by $N q V$*. But it has been shown in Art. 26 that $q = 2\pi^2 m r^2 n^2$ and $N q V$ is therefore equal to $N V. 2\pi^2 m r^2 n^2$ or $2\pi^2 N m r^2 n^2 V$. In this result Nm is obviously the mass per unit volume or *density* of the medium and $V = n\lambda$; the energy transmitted per second across unit area of the surface, that is, the intensity of the disturbance, is therefore given by $2\pi^2 n^3 r^2 \lambda d$. Hence, if n, λ, and d are constant the only

* With a *plane* wave front this result is at once apparent, for in one second unit area of the wave front meets N V molecules.

variable is r, the amplitude of vibration of the medium particles, and if the intensity varies inversely as the square of the distance, then r must vary inversely as the distance from the source.

47. Relation of the intensity of the wave motion at any point to the density of the medium at the centre of disturbance. If the density of the medium adjacent to the source is very low then only a very small quantity of energy is communicated by the source to the medium at each vibration, but if the density is high then a large quantity of energy will be communicated to the medium at each vibration and the intensity of the resulting disturbance will be greatly increased.

The energy communicated by the vibrating spherical source during one complete vibration to the surrounding medium is measured by $2\,\pi^2 n^2 r^2 \lambda\, d$ A, where A denotes the area of the surface of the source, d the density of the medium in contact with the source, and r the amplitude of the vibration of the particles in this layer of the medium. For a given source the value of r for particles of the medium adjacent to the source is practically the amplitude of vibration of the source, hence the energy communicated to a medium at each vibration is, for a source of given amplitude of vibration, directly proportional to the density of the medium adjacent to the source.

This implies that the intensity of a disturbance travelling out through a continuous medium from the source of disturbance, while it obeys the law of inverse squares for distance, is, at any point, directly proportional to the density of the medium at the point where the disturbance originates.

48. Loss of energy by a vibrating body acting as a source of wave motion in a medium. A vibrating body, acting as a source of wave motion in the medium around it, evidently loses energy by the communication of its energy of vibration to the medium as energy of wave motion. The amount of energy communicated to the medium during each complete vibration is, as explained above, measured by $2\,\pi^2 n^2 r^2 \lambda\ d$ A, where d is the density of the medium, and r is the amplitude of vibration of the source.

If, therefore, any body be set in vibration in a medium and allowed to come to rest it loses energy during vibration not only by the dissipation of energy by work done against internal molecular friction, as explained in Art. 25, but also by communication of energy of wave motion to the surrounding medium.

The loss in this way is proportional to the density of the medium and to the square of the amplitude of vibration. A given vibrating body will therefore come to rest more quickly in a dense than in a rare medium, and as the vibration dies away, and the amplitude decreases, the rate of loss of energy by communication to the medium also decreases, being always directly proportional to the square of the amplitude of vibration at any instant.

This communication of energy from a vibrating source to the surrounding medium as energy of wave motion is the most general case of the process known as *Radiation*.

49. Dissipation of energy of wave motion. The energy of wave motion in any medium is ultimately dissipated as heat in the medium, the heat being developed by the work done during the motion against the internal molecular friction which constitutes the viscosity of the medium. The rate of dissipation of the energy is therefore greater the greater the viscosity of the medium.

50. Absorption of energy of wave motion. Energy of wave motion in a medium may be "absorbed" from the medium and converted into energy of vibration in a vibrating body by any body capable of being set in vibration by the impact of the wave motion on it. If a body is free to vibrate, and if its natural frequency of vibration is the same as that of the source of the wave motion, it is capable of being set in vibration by the impact of the wave motion. The wave motion transmits the vibratory motion of the source from layer to layer through the medium, so that when the waves impinge on the body they tend to set it in vibration with the

frequency of the source. But as this is the natural frequency of vibration for the body, the impulse derived from the impact of the waves at any instant is always appropriate in magnitude and direction to the phase of the body at that instant. In this way the impact of the wave motion sets the body in vibration. The initial displacement is almost inappreciable, and the amplitude of the vibration is at first very small. As the body "absorbs" energy from the medium, however, the amplitude gradually increases until the maximum value possible under the existing conditions is attained.

This process of exciting the vibration of a body by the action of waves of the same period as the body is another example of the principle of *resonance* referred to in Art. 24.

EXERCISES VIII.

1. Define the *intensity* of the wave motion at any point and prove that the intensity at a point is inversely proportional to the square of the distance of that point from the source.

2. Prove that the amplitude of vibration of a wave system at any given point is inversely proportional to the distance of that point from the source of the waves.

3. The amplitude of vibration of a particle 100 metres from a source is 1 cm. Find the amplitude at 200 metres. Compare also the intensities at these two distances.

4. In what way does the intensity of the wave motion depend upon the density of the medium surrounding the source?

5. How does a body vibrating in a medium lose energy?

6. Describe how a body may be set in vibration by a series of waves given out by another body. In order that resonance may occur, what must be the relation between the frequency of the waves and the free vibration frequency of the body on which the waves fall?

CHAPTER IX.

REFLECTION AND REFRACTION OF WAVE MOTION.

51. Reflection of wave motion. Wave motion in a medium is reflected when it is incident on the surface of separation of the medium from another medium or substance of different density. Thus longitudinal wave motion in air is reflected at the surface of a brick wall or at the surface of separation of air and water.

If the surface is smooth and plane the incident waves are regularly reflected at the surface in accordance with the laws of regular reflection at a plane surface. If the surface is smooth and curved the waves are reflected at any point of the surface as by a small *plane* area tangential to the surface at that point. If the surface is not smooth the waves are irregularly reflected, and *diffused* or *scattered* at the surface.

The degree of smoothness necessary to determine regular reflection at any surface depends upon the wave length of the incident waves. If the inequalities of the surface are small compared with the wave lengths, regular reflection will take place at the surface. Thus the surface of an ordinary stone or brick wall will act as a "smooth" plane reflecting surface for longitudinal waves in air of, say, more than one foot in wave length.

The *extent* of surface necessary to produce appreciable reflection also depends upon the wave length. The area of the surface should be fairly large compared with the wave length of the incident waves. Thus a smooth plane surface

a square yard in extent will not act as an efficient reflector for waves of more than a few inches wave length. When a surface, such as a rock or a cliff, consists of a large number of small plane surfaces irregularly distributed it may evidently diffuse or scatter waves of large wave length and reflect, at the small surfaces, waves of short wave length.

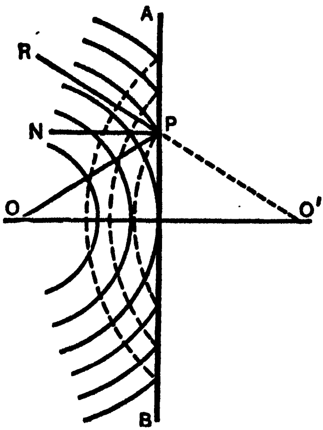


Fig. 38.

Fig. 38 shows diagrammatically the process of regular reflection of a spherical wave at a plane surface. The incident wave diverges from a centre O, and the reflected wave, indicated by the dotted arcs, diverges as if from a centre O′ at the same distance behind the reflecting surface as O is in front of it.

In this way the incident disturbance travelling along any line O P is reflected from the surface along P R so that the angle of reflection N P R is equal to the angle of incidence O P N, and the two lines O P and P R are in the same plane as N P, the normal to the surface at N.

52. Reflection of longitudinal wave motion at a plane rigid surface or at the plane surface of a denser medium. When longitudinal wave motion is reflected at a rigid reflecting surface or at the surface of separation of the medium from a much denser medium, reversal of displacement always takes place at the point of incidence and reflection. Thus let M N (Fig. 39) represent a line of particles in the medium at right angles to the reflecting surface A B, and let *b* be the last particle in the

Fig. 39.

line at N, and *a* the first particle in the reflecting surface. When a longitudinal wave travelling along M N reaches the particle *b* this particle attempts to transmit the forward displacement to *a*. The particle *a*, however, is fixed or only slightly displaceable, according as A B is a rigid surface or the surface of a dense medium, and *b*, in

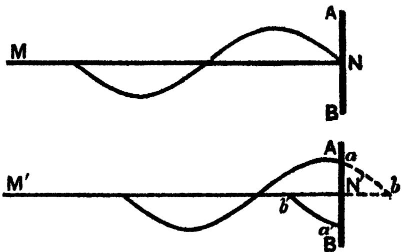


Fig. 40.

attempting to displace *a forwards*, is itself, by reaction, displaced *backwards*, and so originates the reflected wave back along N M.

Hence, if the upper curve along M N in Fig. 40 is the curve of displacement of the incident wave at the instant the disturbance reaches the reflecting surface, then the lower curves along M′ N′ may represent the curves of displacement for the incident and reflected waves at an interval rather less than a quarter of a period later. In this interval the portion of wave represented by *a b* has been reflected at A B, and the displacement curve for this reflected portion is given by *a′ b′*. The successive displacements in the incident wave are here shown reversed in the reflected wave.

The state of strain in the wave is not, however, changed by reflection under these conditions. Thus in the curves along N′ M′ in Fig. 40 the states of gradually decreasing compression found from *b* to *a* in the incident wave are found unchanged in the reflected wave from *b′* to *a′*. That is, each state of strain is reflected unchanged at the surface A B, compression being reflected as compression and rarefaction as rarefaction.

Exp. 4. Fix one end of a wave machine spiral and originate a compression at the other end. It will be found that when the compression reaches the fixed end it is *reflected back as a compression.* Similarly, if a rarefaction is transmitted along the spiral to the fixed end, it is there *reflected back as a rarefaction.*

It is obvious that if A B is a smooth and practically rigid surface the incident wave will be almost totally reflected. A small portion of it will probably be diffused or scattered at the surface, and a very small portion may be transmitted on into the substance behind the reflecting surface. If, however, A B is the surface of separation between the medium of the motion and a denser medium, then only a portion of the energy of the incident wave is reflected, and a considerable portion may be transmitted on into the denser medium.

53. Reflection of longitudinal wave motion at the plane surface of a rarer medium. In the case where longitudinal wave motion is reflected at the surface of separation of the medium of the motion from a *less dense* medium there is no reversal of displacement at reflection; but, as a consequence of this, there is reversal of strain, compression being reflected as rarefaction and rarefaction as compression. If A B in Fig. 39 represents the surface of separation in this case, the particle *b* in attempting to displace the particle *a* forwards meets with little resistance, and consequently undergoes at any instant a greater *forward* displacement than that communicated to it directly by the incident wave. The additional forward displacements thus impressed on *b* are transmitted back along N M, and constitute the reflected wave.

Exp. 5. It can be seen by experimenting with a wave spiral that if both ends of the spiral are free a *compression* (or rarefaction) originated at one end is reflected at the other *free end* as a *rarefaction* (or compression).

The curves along M′ N′ in Fig. 41 represent, as explained with reference to Fig. 40, the curves of displacement for the incident and reflected waves for reflection at the surface of a rare medium at an instant rather less than a quarter of

a period after the first incidence of the wave on A B. In this interval of time the portion of the wave represented by $a\,b$ has been reflected at A B, and the displacement curve for this portion when reflected is given by $a'\,b'$. The successive displacements in the incident wave are not reversed in the reflected portion, but they are diminished

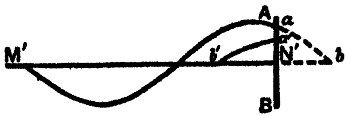


Fig. 41.

in magnitude as, under the conditions considered, the reflection can only be partial. The state of strain is, however, reversed by the reflection, for the phases of gradually decreasing compression found from b to a in the incident wave, become the phases of gradually decreasing rarefaction found from b' to a' in the reflected wave.

In this case of reflection at the surface of a rare medium it is evident that only a small portion of the energy of the incident wave will be reflected. The main portion of it is transmitted on into the rare medium.

Exp. 6. Take two wave machine spirals (of the same diameter) one of which is made of thicker wire than the other, and suspend them so that they hang in the same horizontal line, end to end. Tie the turns at the adjacent ends of the spiral firmly together with thread, so as to make a secure junction between the two spirals. This junction represents the surface of separation of two media represented by the two spirals, the denser medium being represented by the spiral made with the thicker wire.

With this compound spiral the reflection of longitudinal wave motion at the surface of a denser or rarer medium can be easily studied. A *compression* (or rarefaction) originated at the free end of the light spiral travels along it up to the junction, where it is in part reflected back as a *compression* (or rarefaction) and in part transmitted on along the heavier spiral. Similarly, a *compression* (or rarefaction) originated at the free end of the heavier spiral is in part reflected at the junction as a *rarefaction* (or compression) and in part travels on along the lighter spiral.

54. Refraction of wave motion. When wave motion passes from one medium into another in which the velocity of propagation is different, the motion is said to be *refracted* or to suffer *refraction* at the surface of separation of the media.

Refraction takes place as the wave front crosses the surface of separation, and the nature of the process will be best understood by following the passage of a small portion of a wave front across the surface of separation of two different media.

Fig. 42 represents the refraction of a very small portion of the wave front of a spherical wave at the surface of separation between two media M and N, for which the velocity of the wave motion is greater in N than in M. Let A B* represent a small portion of the incident wave front at the instant the disturbance reaches the separating surface at A. During the time the incident disturbance along O B travels, in the medium M, from B to b, the disturbance from A travels in the medium N to all points on the surface of a small sphere round A as centre. The radius of this small sphere must evidently, in this case, be greater than B b, for the velocity of the disturbance in the medium N is supposed to be greater than in the medium M. The refracted wave front in the medium N at the instant the incident disturbance reaches b must therefore be represented by ab, drawn through b and tangential to the small spherical surface at A.

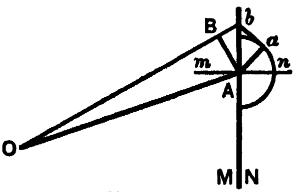


Fig. 42.

The refracted wave front $a\,b$ is evidently not parallel to A B the incident wave front. The distance A a, described by the disturbance from A in the medium N, is greater than the corresponding distance B b described *in the same time* in the medium M by the disturbance from B, and as a result the refracted wave front is tilted forward at a so as to make a small angle with A B the incident wave front. This change of front, due to the difference of velocity in the media, is characteristic of refraction when the incident wave front makes an angle with the surface of separation. When, however, the incident wave front is parallel to the surface of separation, there is no change of front, for the refracted wave front is parallel to the incident wave front, but travels on in the new medium with the velocity of wave motion in that medium.

Index of Refraction. In Fig. 42 the wave front element A B is incident on the separating surface at A, at the angle B A b, and the

* A B may be considered to represent a central section, in the plane of the paper, of a very narrow zone of the spherical wave front diverging from O. In the same way $a\,b$ may represent a section of the corresponding zone of the refracted wave.

refracted wave front element $a b$ makes an angle $a b A$ with the separating surface. These angles are called respectively the *angle of incidence* and the *angle of refraction*, and, in this case, where the velocity in medium N is greater than in medium M, the distance $A a$ is greater than $B b$, and the angle of refraction greater than the angle of incidence. Further, since $A B$ is a very small element of a spherical wave front, we may consider $A B$ and $a b$ as straight lines, and the triangles $A B b$ and $A a b$ as right-angled triangles. Hence it follows that $\sin B A b = b B / A b$ and $\sin a b A = A a / A b$, and the ratio $\sin B A b / \sin a b A = \dfrac{b B}{A a}$. But $A a$ and $b B$ are the distances travelled by the disturbance *in the same time*, in the media N and M respectively, and therefore

$$b B / A a = V_M / V_N$$

where V_M and V_N are the velocities of the disturbance in the media M and N. That is,

$$\sin B A b / \sin a b A = V_M / V_N .$$

We thus have the result that when refraction takes place at any point from one medium M to another N the ratio of the sine of the angle of incidence to the sine of the angle of refraction is constant and equal to the ratio of the velocity of the wave motion in the medium M to the velocity in the medium N. This ratio is usually called the *index of refraction*, and, for a given disturbance, is a constant for any two media whatever the values of the angles of incidence and refraction.

If, in Fig. 42, we consider the linear disturbance that travels along $O A$ as an incident "ray" or line of disturbance, then $A a$ is the corresponding refracted "ray," and if $m n$ be the normal at A, the point of incidence, then $O A m$ and $a A n$ are taken as the angles of incidence and refraction for these rays. They are respectively equal to the angles $B A b$ and $a b A$, and the ratio of their sines determines the index of refraction for the media. The usual law of refraction states that the ratio of the sines of these angles is constant and that the incident and refracted rays and the normal are in the same plane. The incident ray $O A$ is refracted along $A a$ and suffers a *change of direction* characteristic of refraction. This change of direction corresponds to and is equal to the change of front referred to above. It is obviously zero when the incident ray is normal to the surface of separation of the media.

Refraction of a spherical wave at a plane surface of separation between two media. In order to follow the process of refraction of a spherical wave at a plane surface of separation between two media it is convenient to divide the surface up into small zonal elements, such as $A B$ in Fig. 42, and to follow the refraction and subsequent transmission of each element as it reaches the surface of separation.

In Fig. 43 the incident spherical wave diverges from O in the medium M. As the wave advances on to the separating surface, the elements of wave front 1, 2, 3, 4 are refracted in succession as they reach the surface, and each new element of the refracted wave front as it is formed advances into the medium N in a direction normal to it with the velocity of the disturbance in that medium. In this way, as the elements marked 1, 2, 3, 4 are refracted in succession, the refracted wave fronts indicated by the dotted curves 1, 2, 3, 4 are formed in the medium N.

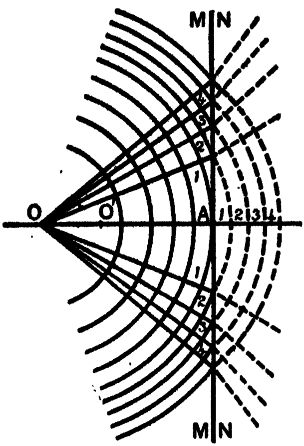


Fig. 43.

The refracted wave fronts obtained in this way are not spherical. If, however, we consider the refraction to take place only at a small area of the separating surface, taken round A, the foot of the perpendicular from O on to the surface, it can be shown that the refracted wave is approximately spherical and diverges from a point O′ on the line O A, such that the ratio of the distance O′A to O A is equal to the ratio of the velocity in the medium M to the velocity in the medium N. That is, the ratio O′A to O A is equal to the index of refraction from the medium M to the medium N.

55. Partial reflection and refraction. From what has been said above on the subjects of reflection and refraction of wave motion it will be understood that whenever wave motion is incident on a surface of separation between two media both reflection and refraction take place, and the incident wave gives rise to a reflected wave travelling back into the first medium and a refracted wave travelling on into the second medium. The sum of the energy of the reflected and refracted waves is equal to the energy of the incident wave, provided the loss by diffusion or irregular reflection at the surface of separation is negligibly small.

EXERCISES IX.

1. Explain why a brick wall will reflect waves of sound, but not waves of light.

2. Explain why a plane mirror one inch square will reflect waves of light, but not waves of sound.

3. Describe the process of reflection of longitudinal wave motion in a medium at the surface of (*a*) a denser medium, (*b*) a rarer medium.

4. In the case of reflection of longitudinal waves at the surface of a very dense medium most of the energy in the incident wave is reflected, while in the case of reflection at the surface of a very rare medium most of the energy is transmitted. Why is this?

5. Explain by a diagram how refraction takes place at the separating surface of two different media.

6. Show that the laws of reflection and refraction of longitudinal waves are the same as the laws of reflection and refraction of waves of light.

7. The velocity of longitudinal waves in hydrogen is 1280 metres per second and in oxygen 320 metres per second. Find the index of refraction between oxygen and hydrogen for these waves.

CHAPTER X.

INTERFERENCE OF WAVE MOTION.

56. Interference of wave motion. When wave motion from more than one source is travelling through any medium, the disturbance (displacement and strain) at any point in the medium is, at any instant, the *resultant* of the individual component disturbances reaching the point from the several sources at that instant. That is, the different disturbances reaching any point in the medium, at a given instant, are *superposed* or *compounded*, and the actual disturbance at the point is the resultant of these component disturbances. This is the principle of *interference* as applied to wave motion.

57. Interference between two longitudinal waves of the same period and amplitude travelling in opposite directions along the same line. As a simple case of interference imagine a longitudinal wave disturbance, travelling along A B from A to B, Fig. 44, and meeting a similar disturbance of the same wave length and amplitude travelling along B A from B to A. When the disturbances meet interference will take place and the motion of the particles affected by both disturbances will, at any point, be the resultant of the motions due to the disturbances taken individually.

The general character of this resultant motion may be determined by drawing the displacement curves of the two disturbances along A B and B A and compounding these curves graphically, by taking the algebraic sum of these ordinates, to obtain the displacement curve for the resultant motion at any instant.

In Fig. 44 let A *a* and B *b* represent the displacement curves for a complete wave length of the two disturbances. Since the waves travel with the same velocity they will meet at a point N halfway between *a* and *b*; this is shown in Fig. 45.

Fig. 44.

When the waves meet, the two disturbances reaching the particle at the point of meeting must obviously be either in the same phase, or in opposite phases, that is, in phases differing by half a period; the particle is at rest when the disturbances reach it, and these disturbances must tend to displace it either in the same direction or in opposite directions. As the curves are drawn in Figs. 44-46, the front points *a* and *b* are in opposite phases, and as the waves meet, the disturbances at the meeting point will therefore always be in opposite phases. The displacement at this point will therefore always be zero.

Fig. 45.

Let the waves now travel on for three half-periods, and from the component curves determine the resultant displacement curves in the region of interference at intervals of one-eighth of a period from the beginning to the end of this time. These curves are shown from 1 to 12 in Fig. 46. The component displacement curves for the two simple waves travelling in opposite directions are drawn in thin lines, and the resultant displacement curves in the region of interference in thicker lines. It will be seen that the component curves in the sequence 1 to 12 advance, with the waves they represent, by one-eighth of a wave length during each interval of one-eighth of a period.

During the first half-period the region of interference extends over a complete wave length, L M, and the gradual development of the resultant motion in this region is shown by the resultant displacement curves from 1 to 4 in the figure. The motion in this length L M is evidently completely representative of the motion in the region of interference, for it will be seen that the resultant displacement curve in this region, as it extends, is merely a repetition of the section for L M. The resultant displacement curve in 4 shows, however, only the displacement in the line L M at a *particular instant*, the end of the first period. In order to get a complete idea of the motion in the region LM, it is necessary to follow the displacement curves for this region for *a complete period.*

These are given by the curves 4 to 12, of which 4 to 8 apply to the second half-period and 8 to 12 to the third half-period of the three half-period intervals under consideration.

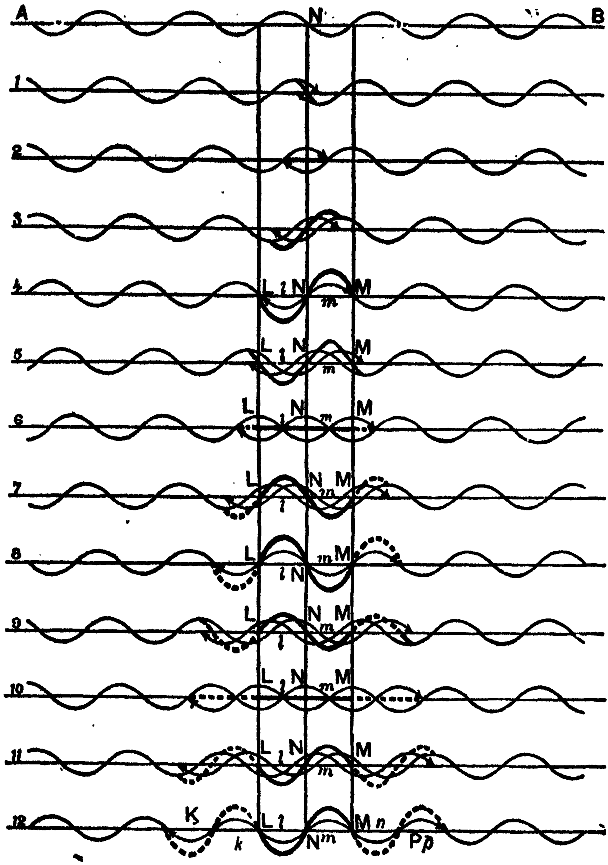


Fig. 46.

A study of these curves will show that the motion in the region of interference is very different from the

If the wave is incident on the reflecting surface A B (Fig. 47) along the normal M N, the reflected wave will be reflected back along N M, and stationary wave motion will be set up as the result of interference between the incident and reflected waves along this line. The point N is the meeting point of the two interfering waves, and as it must be a node, there must be, as explained in

Fig. 47.

Art. 52, a reversal of displacement at reflection. The conditions of interference are therefore exactly similar to those dealt with above, and to which Fig. 46 applies. The waves have the same wave length, nearly the same amplitude, and the difference of phase at the meeting point is half a period.

When the stationary wave is established along M N, nodes will be found at equidistant points N, *n*, *n*, *n* along N M, and the length of each internode will be one-half the wave length of the incident wave.

If there happens to be much loss of energy at reflection, the amplitude of vibration in the reflected wave may be appreciably less than in the incident wave. In this case the general results of interference will be unaltered, but the characteristics of the stationary wave will not be so well marked. The nodes, for example, instead of being points of *zero* displacement, will be points of minimum range of displacement or minimum amplitude of vibration.

If we imagine the reflecting surface to be at N, Fig. 46 will apply also to this case of interference. The wave travelling from A to B may be taken to represent the incident wave, and the wave travelling from B to A the reflected wave.

EXERCISES X.

1. Explain what is meant by *interference* as applied to wave motion.

2. Describe *stationary wave motion.* Explain how stationary wave motion may result from the *interference* of travelling waves.

3. Describe experiments with a wave machine illustrating (*a*) the transmission of a pulse of condensation or rarefaction, (*b*) the reflection of a pulse of compression or rarefaction, (*c*) stationary wave motion.

CHAPTER XI.

SOUND.

60. Sound. The term Sound may apply to the sensation of sound received through the ear or to the external physical causes of this sensation. The science of sound, as a branch of physics, deals mainly with the external physical causes to which the sensation is due. The study of the sensation itself involves both Physics and Physiology. The sensation of sound is, in general, produced by the impact on the drum of the ear of any disturbance in the air which involves a sufficiently rapid, but not too rapid, sequence of changes in the pressure of the air adjacent to this membrane. These changes of pressure, acting on the thin stretched membrane which constitutes the drum, set it in motion, inwards and outwards, responsive to the character of the disturbance incident upon it. This motion when communicated to the *inner ear* produces the sensation of sound.

61. Nature of sound. It is a matter of common experience that a body in rapid vibration is a source of sound. A stretched string when set in vibration emits a sound; when the vibration is stopped or allowed to die away the sound is no longer heard or it becomes fainter and fainter. An ordinary tuning fork is made to sound by setting its prongs in rapid transverse vibration, and it continues to be a source of sound so long as the vibration lasts.

Exp. 7. Set a bell jar, such as is shown in Fig. 48, in vibration by bowing its edge with a violin bow. The jar at once emits a clear sound which is at first fairly loud, but becomes fainter and fainter as the vibration decreases in amplitude. If the vibratory motion is stopped, say, by laying the hand on the side of the jar, the sound at once ceases.

When the jar is first bowed, and is sounding strongly, the vibratory motion may be detected by noting the blurred outline of the rim. When, however, the sound is faint the vibration may not be visible but can be at once detected by bringing a pith ball suspended by a fine thread, or the end of a loosely held pointer, in contact with the side of the jar.

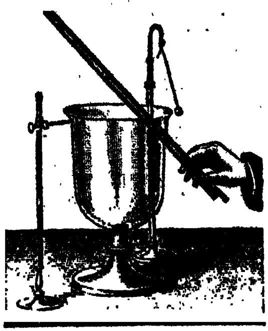


Fig. 48.

Exp. 8. Sound a tuning fork, and while it is still sounding faintly, but apparently not vibrating, touch the tongue or a sheet of glass with the end of one of the prongs. The result will show that the prongs are in rapid vibratory motion of very small amplitude.

Although a vibrating body is, in general, a source of sound, this is not always the case. It is found that if the frequency of vibration falls below a limit of about 30 per second no sound is heard. Also, if the frequency rises above a limit, set variably at from 20,000 to 40,000 vibrations per second, no sound is heard. These limitations are obviously limitations of sensation determined by the structure of the ear.

Exp. 9. Fix a fairly long steel strip in a vice as shown in Fig. 10, and adjust the free length so that the frequency of vibration is about ten or twelve vibrations per second. No sound is heard when the strip is in vibration. Now shorten the free length, step by step, and set the strip in vibration after each shortening. It will be noticed that as the frequency increases and approaches the lower limit of audibility a dull whirring sound is first *felt*, rather than heard, and ultimately a continuous note of very low pitch is produced when the frequency has reached the limit of about 30 vibrations per second.

These facts suggest that the sensation of sound is due to the incidence on the drum of the ear of the longitudinal wave motion set up in the air by a body in vibration within certain limits of frequency. That is, the vibrating body is the *source* of the sound. The vibratory motion of the source sets up longitudinal wave motion in the air and the incidence of this wave motion on the ear causes the sensation of sound.

The science of sound must therefore concern itself with (*a*) the vibration of bodies, for a vibrating body is a source of sound and, as such, is sometimes called a *sounding body* or a *sonorous body*; (*b*) the propagation of longitudinal wave motion in the air and other media, for longitudinal waves in any material medium are *sound waves*; and (*c*) the sensation of sound. Sound, as a branch of Physics, deals mainly with (*a*) and (*b*).

In a limited sense sound may be defined as longitudinal wave motion in a material medium, just as radiation may be defined as transverse wave motion in the ether.

62. The ear. The detailed study of the ear as the organ for the perception of sound belongs to Physiology. It should, however, be noted that the structure of the outer ear confirms the view that the sensation of sound is caused by the incidence of longitudinal waves on the drum of the ear. The drum or *membrana tympani* is stretched across the tube leading from the external orifice to the inner ear and divides it into the *external meatus* and the *internal meatus*. It communicates with the inner ear by a chain of three bones, one end of the chain being attached to the inner surface of the drum membrane, and the other end to the membrane closing the *fenestra ovalis*, an opening in the bony wall of the inner ear. The external meatus communicates directly with the outer air, and the internal meatus can also be put in communication with the air through the Eustachian tube leading to the mouth.

When longitudinal wave motion enters the external meatus and falls upon the drum of the ear it sets this membrane in forced vibration. This vibration is communicated through the chain of bones to the inner ear,

where it reaches the auditory nerves and produces the sensation of sound.

A study of the structure and action of the drum of the ear shows that it is well adapted to receive and respond to longitudinal wave motion, and that it probably can, to a limited extent, be adjusted in sensitiveness for response to waves of different types. It should be noticed that the drum responds, not to the displacement of the air particles in the incident wave, but to the changes of pressure which attend the cycle of compressions and rarefactions which constitute a longitudinal wave. When the air adjacent to the outer surface of the drum is compressed the *increase of pressure* forces the membrane inwards, and when the air is rarefied the *decrease of pressure* allows the membrane to move outwards. The amplitude of motion of the drum is however very small, probably not more than a tenth of a millimetre at most, and as small as a millionth or a ten-millionth of a millimetre for very faint but audible sounds.

63. A musical sound. It will be recognised that the sound emitted by bodies in regular vibration are all of the particular type known as *musical sounds*. For example, the sounds produced by the vibration of the prongs of a tuning fork, the strings of a violin, the steel wires of a piano, and the columns of air in the pipes of an organ are all musical sounds. This implies that the incidence on the ear of longitudinal waves of well-marked periodicity and definite constant wave length gives rise to the more or less pleasant sensation associated with what is called a musical sound or a musical note.

The essential element of a musical sound is periodicity. The motion of the source must be periodic and the waves of constant wave length, every wave length being of the same type as regards sequence of states of compression and rarefaction. In all cases where this element of periodicity obtains the sound is found to be "musical." There may be differences in the pitch or quality of the sound depending on the wave length or on the type of the waves, but the sensation produced is always that associated

with a musical note. When the source of sound is a vibrating body the element of periodicity is *the* characteristic of the motion of the source and of the wave motion. The sound produced by a vibrating body is therefore always a musical sound. If the body vibrates in true simple harmonic motion the wave motion set up is simple harmonic wave motion and the sound produced is a musical sound of a special quality known as a *pure* tone. If the body vibrates in a more complex way the wave motion is of a correspondingly complex type, but the sound produced is still a musical sound though not of pure tone.

A musical sound may, however, be produced by sources other than vibrating bodies. Any source capable of originating a *periodic* disturbance in the air may produce a musical sound. Thus if a card or thin strip of wood be held so as to be struck by the teeth of a toothed wheel rotating sufficiently rapidly at a uniform speed a distinct musical note is produced. The *hum* of a circular saw at work is an example of a sound produced in this way. The motion of the card produced by the rapid succession of taps from the teeth of the wheel is of a periodic character, and the waves originated by the card will be of constant wave length and similar in type, although this type in the matter of sequence and gradation of states of compression and rarefaction may be of a somewhat irregular character. Similarly, if a stream of air from a nozzle is interrupted periodically, say by means of a rotating disc pierced with a circle of equidistant holes, the sound produced is a musical note. The puffs of air following each other at regular intervals originate waves of constant wave length and fixed type and produce a musical sound of a very characteristic quality.

Exp. 10. Arrange a toothed wheel with equidistant saw teeth so that it can be rotated rapidly at a uniform speed, as shown in Fig. 49. Fix a card, as shown in the figure, so that it is lightly struck by the teeth of the rotating wheel. If the wheel is rotated sufficiently rapidly it will be found that a distinct musical note is produced.

A toothed wheel for this purpose was first used by Savart and is therefore commonly called Savart's toothed wheel.

Exp. 11. Take a circular plate of cardboard or metal, pierced with a circle of equidistant holes, as shown in Fig. 50, and mount it, like the Savart wheel in Fig. 49, so that it can be rotated rapidly round an axis passing through the centre of the circle of holes and at right angles to the plate. Direct a stream of air, forced through a nozzle against the ring of holes, as shown in Fig. 54. If the plate is rotated rapidly enough the rapid sequence of puffs of air through the holes in the plate set up a periodic wave disturbance in the air and a musical sound is heard.

This simple apparatus for producing a musical sound is known as the *cardboard siren* or *Seebeck's siren*. It should be noticed that an important characteristic of this apparatus and of Savart's wheel is that the *frequency* of the originating disturbance can be calculated directly if the speed of revolution is known. Thus, if the number of teeth or holes is 50 and the speed of revolution is 10 per second, the frequency is evidently 500 per second.

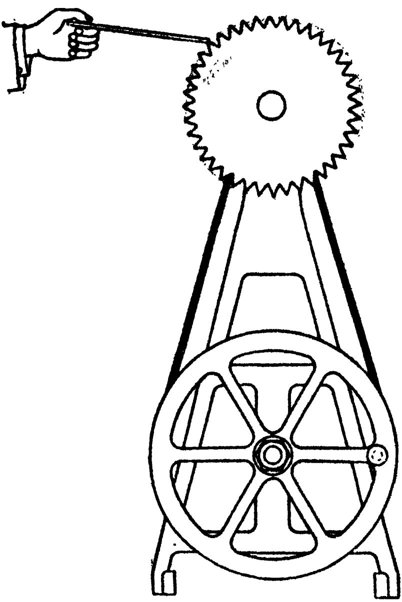


Fig. 49.

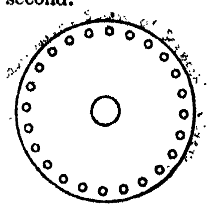


Fig. 50.

64. Noise. All the sounds we hear are, however, not musical sounds. Sounds other than musical sounds vary greatly in character, but may all be included under the general term *noise*. The essential characteristic of a *noise*, as distinguished from a musical sound, is irregularity or *want of periodicity*. Any source in *non-periodic* motion

giving rise to an irregular wave disturbance with sufficiently rapid variations of pressure may produce a noise of a character determined by the nature of the disturbance which reaches the ear. The wave disturbance associated with a noise will be of no definite wave length, the sequence of states of condensation and rarefaction which constitute the disturbance will follow no regular law, and, as there is no definite wave length, there will be no recurrence of a definite cycle of states, wave length after wave length, as in the case of the periodic wave associated with a musical sound.

It should be noticed that the originating disturbances in the case of most noises are not the large and sometimes violent air displacements which accompany the production of the sound, but very rapid movements of very small amplitude or rapid changes of pressure produced somewhere at the source of the sound. Thus, when a piece of wood is struck with a hammer, the source of sound is not the air displacement produced by the moving hammer, but the rapid irregular vibration of the surface particles of the wood, caused by the stroke of the hammer. The sound produced by clapping the hands is caused, not by the motion of the hands in the air, but by the sudden expulsion of the air from between the hands at the instant of contact. The noise of an explosion is produced, not by the violent displacement of the air at the centre of explosion (the "wind" of the explosion), but by the compressions and rarefactions communicated to the surrounding air by the sudden changes of volume in the explosion region. The sound of thunder is caused by the sudden expansion and the subsequent sudden contraction of the intensely heated air along the track of the lightning flash. All these sources of disturbances, however irregular they may be in action, set up wave disturbances of irregular non-periodic type in the air, and the incidence of these disturbances on the drum of the ear gives rise to sound sensations or noises characteristic of the disturbances.

The difference between a musical sound and a noise is well exhibited by means of the displacement or strain curves of the wave

of sound. This law is subject to the limitations explained in Art. **45**. If the source is very large, or if the originating disturbance involves a large volume of the medium, as in the case of the discharge of a cannon, the loudness of the sound to a person near the source, where the wave front is far from spherical, varies with position rather than with distance. When, however, the distance from the source is very large compared with the dimensions of the source, the loudness of the sound varies inversely as the square of the distance from the source.

3. The loudness of a sound varies with the density of the medium in the neighbourhood of the source. The reason for this is explained in Art. **47**, where it is shown that the intensity of the wave is proportional to the density of the medium at the source.

Exp. 13. Suspend a small bell in a large flask fitted so that it can be filled, as required, with air, water vapour, coal gas, hydrogen, carbon dioxide, or other gas. When the bell is shaken it will be found that the loudness of the sound heard varies with the density of the gas, being greater the greater the density.

If two observers at different levels, one, say, on the top of a mountain and the other in the valley below, each fire a rifle of the same pattern, the sound heard by the observer in the valley will be much less intense than that heard by the observer on the mountain top.

The loudness of the sound emitted by a vibrating body may in some cases be greatly increased by the action of a *sounding board.** When a tuning fork, for example, is sounding, the loudness of the sound emitted may be very greatly increased by resting the end of the fork handle on a piece of wood, such as a light table top or box cover. This is explained by the fact that the vibration of the fork sets up, as explained in Art. **24**, forced vibration in the wood particles, and the surface over which energy is communicated from the fork to the medium being thus greatly increased, the intensity of the wave motion at any point in

* The use of a sounding *box* involves also the principle of resonance. See Art. **24**.

the medium is proportionately increased and the sound greatly intensified. The duration of the sound is, however, proportionately decreased, for the initial energy of the vibrating fork is approximately constant and the rate of radiation of energy is greatly increased.

67. Pitch. The pitch of a musical sound is found to depend on the *frequency* of the source of the sound.

Exp. 14. Set a "cardboard" siren and a Savart wheel in action and arrange so that the speed of revolution can be easily varied. It will be found both with the wheel and the siren that when the frequency changes the pitch changes. When the frequency is increased the pitch rises, when it is decreased the pitch falls, and when it is constant the pitch is constant.

The relation between pitch and frequency is very satisfactorily illustrated by the apparatus* shown in Fig. 54. Four Savart wheels, W, and a siren plate, D, pierced with four concentric rings of equidistant holes are fitted on the same spindle and geared so as to be rotated at any required speed by means of the driving wheel shown in the figure. The number of teeth on the wheels and the number of holes in the rings of the siren plate are in the ratio 4 : 5 : 6 : 8, the number of teeth on any one wheel being the same as the number of holes in the corresponding ring of holes in the plate. A card C is arranged so that it can be adjusted to any one of the four wheels, and the tube T, through which a jet of air can be directed on to the siren plate, is fixed so that it can be adjusted to any ring of holes in the plate.

Exp. 15. With this apparatus adjust the card to *any one* of the toothed wheels and the jet of air to the corresponding ring of holes. It will then be found that at whatever speed the apparatus may be revolved the pitch of the two notes produced is always the same, although the quality of the notes is very different. It will also be noted, as in Experiment 14, that as the speed of revolution is varied the pitch for each note rises and falls in exactly the same way. That is, so long as the frequency is the same for two notes, however differently they may be produced, the pitch is the same.

* See Poynting and Thomson's *Sound*, page 9.

Exp. 16. With the same apparatus, while the speed of revolution is practically constant, touch each of the four wheels in rapid succession with the card, beginning with the one with the smallest number of teeth. The succession of notes produced will be at once recognised as the common chord for the note of lowest pitch. In the same way, if the jet of air is directed in rapid succession against each of the four rings of holes, the same sequence of notes will be produced.

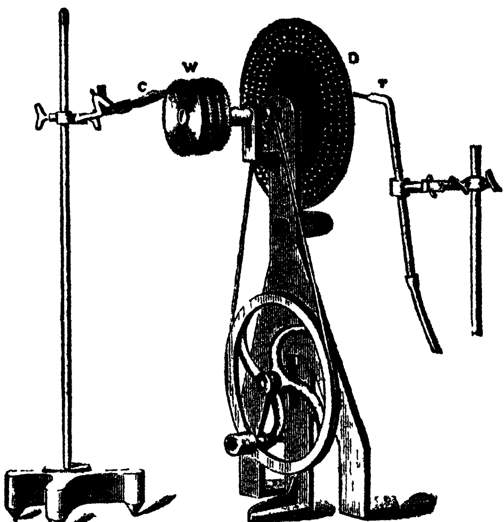


Fig. 54.

This result will be found to be the same at all speeds. The actual pitch of each note will be different at different speeds, but the *intervals* between the successive notes will remain constant. This shows that while the pitch of any note is determined by the frequency of its source the *interval* between any two notes is determined by the *ratio* of their frequencies. Thus, from the result of the experiment for pairs of notes whose frequencies are in the ratios 4 : 5, 4 : 6, and 4 : 8, the corresponding intervals are, at all speeds, the major third, the fifth, and the octave which

occur in the common chord. These ratios and the corresponding intervals should be noted, particularly for the octave. It will be observed that when the frequency of a note is doubled its pitch is raised by an octave.

It has already been stated that a musical sound cannot be heard unless the frequency is greater than about 30 per second. That is, the lowest limit in pitch for an audible musical sound corresponds to a frequency of about 30 vibrations per second. Experiment shows that there is also an upper limit in pitch for audibility. If the frequency is too high the sound produced cannot be heard by the human ear. This upper limit in pitch is found to vary within somewhat wide limits for different individuals. It has been found by various observers to correspond to frequencies varying from 16,000 to 40,000 vibrations per second. The average value of the limit may be set at a frequency of about 20,000 per second.

Exp. 17. Take a large tuning-fork constructed so that it can be adjusted by sliding weights on the prongs to a frequency varying from 25 to 35 per second. It will be found that when the fork is excited the note is quite inaudible at the lower limit of its range, but distinctly audible at the higher limit. It is somewhat difficult, however, to determine the exact frequency at which audibility occurs.

Exp. 18. If a small whistle, constructed so as to be capable of producing a note of frequency varying from 10,000 to 40,000 vibrations per second, be sounded and the pitch of the note gradually raised, Art. 110, by shortening the vibrating column of air in the barrel of the whistle, it will be found that, at a certain point differing for different observers, the note becomes quite inaudible.

The pitch of a musical note is thus determined by the frequency of the source. When the frequency is constant the pitch is fixed and definite, and when the frequency changes the pitch changes. Pitch is therefore the essential characteristic of a musical sound, for, being determined by the frequency, it is directly associated with the periodicity of the note. A noise, being non-periodic in character and of no definite frequency, cannot be said to have pitch.

It is found, however, that the sensation of pitch is produced by a very slight persistence of periodicity. The incidence of two or three complete wave lengths in direct

succession on the ear is sufficient to establish the sensation of pitch. Hence, if a vibrating body executes only two or three complete vibrations, the sound heard may be a note of well-defined pitch.

Exp. 19. Construct a cardboard siren plate and mark it for five concentric rings of holes with the same number of holes for each ring. Cut out (with a cork borer) two consecutive holes in the innermost ring, four in the next, six in the next, and ten in the outermost ring. Mount the card for rotation, and as it rotates let the jet of air be directed on each of the incomplete rings of holes and notice the pitch of the notes produced. It will be found that, at any given speed, the pitch is the same for each ring, and that, while the pitch is more definite the greater the number of holes, it is fairly clearly defined even for the innermost ring, where the periodicity of the note is determined by the sequence of only two complete vibrations.

68. Quality. When a train of similar waves of equal wave length fall upon the ear, the sensation produced is that which we associate with a musical sound. The special character or quality of the sound, as received by the ear, will, however, depend upon the exact character of the sequence of states which constitutes each wave length. In sound waves, that is, in longitudinal waves, the sequence of states which constitute a wave length is always a complete cycle of states of compression and rarefaction, but the manner in which the state of the medium changes from point to point in a wave length may give infinite variety to the character of the same general cycle of changes. The characteristic of a musical sound which is associated with and dependent upon the character of the sequence of states which make up a wave length is known as the *quality* or *timbre* of the sound.

The property of a wave which determines the quality of the sound is most clearly exhibited by means of the displacement or strain curve for the wave.

Figures 55, 56 represent the displacement curves for the waves of two musical sounds of the same pitch but of quite different quality. The wave length is the same for the two curves, but the character of the sequence of states which make up a wave length, *as shown by the form of the curve*, is different. The *quality* of a sound is thus indicated by the *form of the displacement curve* or strain curve for

the wave, and, for this reason, by a transference of terms, the quality of a sound is sometimes said to depend upon the *form* of the wave.

The curve in Fig. 55 represents approximately the sound wave produced by the vibration of a tuning fork. It differs little from the simple sine curve representative of a simple harmonic wave. The curve in Fig. 56 represents the sound wave produced by the vibration of a violin string. The curve is strictly periodic, but differs widely from the simple harmonic form.

Fig. 55.

These two curves differ essentially in *form*, and the two sounds they represent differ essentially in *quality*. The first, represented by a simple harmonic curve, is said to be a *pure* or *simple* note; the second, represented by a curve which is really compounded of a series of simple harmonic curves, is said to be a *compound* or *complex* note compounded of a series of simple notes blended together.

Fig. 56.

It should be noted that pitch, loudness, and quality are *sensations* which may be referred respectively to the frequency, amplitude, and nature of the displacement variation (Art. 16) *of the source of the sound*, or to the frequency, amplitude, and displacement variation *of the particles of the medium carrying the wave motion*, or to the wave length, intensity and wave form *of the wave motion in the medium*. Strictly speaking, however, the sensations of pitch, loudness, and quality depend primarily (as sensations) on what takes place at the outer ear, and should therefore be referred respectively to the number of wave lengths incident upon the drum of the ear per second, the intensity of the wave motion at the drum, and the form of the waves incident on the drum.

EXERCISES XI.

1. When a bell is struck under the receiver of an air-pump, explain why the sound dies away as the receiver is exhausted.

2. What is the essential feature of a musical sound? How does it differ from a noise?

3. What are the three characteristics of musical sounds? How is the movement of the air-particles affected by (*a*) change of pitch, (*b*) change of intensity?

4. A circular disc, pierced with a concentric ring of equidistant holes, rotates about an axle which passes through its centre. A stream of air from a small fixed jet is blown against the holes. On what do the *intensity* and *pitch* of the note produced depend?

5. Explain clearly upon what properties (1) the loudness, (2) the musical pitch, of a note depend.

6. Describe experiments to show (1) that sound is produced by the vibrations of the sounding body; (2) that the loudness of sound depends on the amplitude of the vibrations.

7. A tuning-fork is set in motion, and you hear its note. The sound is conveyed to your ear by the motion of the air particles in the room. Explain how, and in what direction, the particles move, and how the motion would be modified if the fork were made to give a louder note.

8. The humming of insects is caused by the beating of their wings, and the hum of a gnat is much higher than that of a blue-bottle. To what conclusion does this point? Describe an experiment which supports your explanation.

9. Two notes of frequencies 200 and 300 are sounded. What is the *interval* between them?

10. A cog-wheel having 60 teeth is rotated 8 times per second. A card is held against the teeth. What is the frequency of the note heard?

11. A siren plate pierced with 50 holes is rotated at a constant speed and an air jet played upon the holes. A note of frequency 500 is heard. How many revolutions is the plate making per second?

12. The note given by a vibrating string is readily distinguished from one of the same pitch given by a flute. Explain in what respects they differ from each other.

CHAPTER XII.

PROPAGATION OF SOUND.

69. Introductory. Sound is propagated as longitudinal wave motion in material media. The source of sound originates longitudinal waves in the surrounding medium, and these waves are propagated through the intervening medium or media to the ear of the observer, where they excite the sensation of sound. In order that a sound may be heard it is therefore essential that a medium or media capable of transmitting sound waves should extend continuously from the source to the observer. Any material medium which possesses elasticity is capable of transmitting sound waves, and, as explained in Chapter VII., the speed of transmission is determined by the elasticity and density of the medium. It should be noted, however, that an essential condition for the effective propagation of sound in any substance is continuity of structure. Non-continuous materials such as sawdust, wool in bulk, felt, and other similar substances do not possess elasticity proper, and cannot act effectively as *conductors* of sound, although the individual particles or fibres of the material may be of good conducting material. Similarly, a substance of grained structure, such as wood, conducts sound better along the grain than across the grain. In the direction of the grain the continuity of the wood fibres greatly facilitates the transmission of the sound waves.

Illustrations of the transmission of sound by different media are readily found in every-day experience. Sounds produced in air are transmitted through the air to the ear. A light tap at one end of a log of wood is transmitted

through the wood and may be heard distinctly by applying the ear to the other end of the log. Similarly a light tap on a continuous wire, such as telegraph line, or on a long continuous iron pipe or rail, is transmitted along the metal to great distances. With a sufficiently long rail or pipe it is possible to arrange to hear at one end two sounds from a single tap at the other end. One sound is transmitted through the metal rail, the other through the air, and, as the rate of transmission is slower in air than in the metal, the sound transmitted through the air is heard a little later than that travelling through the metal.

A bell rung under water or an explosion under water may be heard distinctly in the air. Similarly, a sound produced in the air may be heard under water. Divers, for example, when under water can hear sounds produced in the air, but as the sound waves are almost totally reflected at the surface of the water only very loud sounds can be heard with any distinctness. Sounds produced under water can be heard under water with great distinctness and at great distances from the source of sound.

Substances are found to differ materially in their power of transmitting sound. Light elastic homogeneous substances in which there is little dissipation of the energy of wave motion are good conductors of sound. Inelastic substances, or substances in which there is rapid dissipation of the energy of wave motion, are bad conductors of sound. Liquids are in general good conductors of sound, unless the viscosity is so high as to cause rapid loss of energy by dissipation.

It is also, in common experience, a noticeable fact that sound does not readily pass from one medium to another of very different density. Thus the sound made by the wheel of a train on the rails of the line is transmitted to great distances along the rails, but it is not audible in the air even quite close to a rail. If, however, the ear be applied directly to the rail, or to the end of a rod of wood touching the rail, the sound of the wheels on the rail is distinctly heard. The wave motion passes directly from the iron to the bones of the head, or from the iron to the wood, and from the wood to the bones of the head, and so

reaches the inner ear. If a square of light wood were fastened to one end of the rod of wood and the other end applied to the rail the sound might become audible in the air near the square of wood.

Similarly the sound of water running in pipes underground is transmitted through the ground and can be heard by applying the ear to the surface of the ground or to the end of a sounding rod resting on the ground. Unless the sound is very loud it cannot, however, be heard in air.

The action of the *stethoscope* used by medical men is another illustration of the same point. The sounds of the lungs and heart in action are transmitted through the surrounding tissues of the body, but not, to any appreciable extent, to the external air. They cannot therefore be heard by placing the ear close to the chest, but if the ear and side of the head rest on the chest wall the sounds can be heard with some distinctness. They are, however, more conveniently and more distinctly heard by means of the stethoscope. In the wooden form of the instrument the sound is conducted to the ear by the wood, and also, and perhaps mainly, by the column of air extending in the tube and ear from the surface of the body to the drum of the ear. In the bi-aural forms of the instrument the sound is conducted to the ear by the air in the cup and tubes of the instrument.

Exp. 20. Suspend a loud ticking clock in the receiver of an air-pump, or support it on a thick pad of loose felt in the receiver. The ticking of the clock can be heard plainly by an observer near it. The sound waves pass through the air and the glass of the receiver to the ear of the observer. Now pump the air out of the receiver and notice the effect on the sound heard. It will be found that as the receiver is exhausted the ticking sound becomes fainter and fainter and finally ceases to be heard when a fairly good vacuum is produced. This shows that the transmission of sound from source to observer is dependent upon the existence of a material conducting medium. When the air is withdrawn from the receiver there is no material medium surrounding the clock in which sound waves can be originated. The clock, as a source of sound, cannot however be effectively insulated; a faint disturbance is transmitted by the felt pad or by the suspension thread, and even at a very good vacuum there is some air present in the receiver.

This experiment also shows incidentally that the loudness of the sound heard depends upon the density of the medium in which it is produced.

Exp. 21. Enclose a clock which ticks loudly in a small box and pack this box with sawdust. If the inner box is completely surrounded by sawdust it will be found that the ticking cannot be heard. Take a piece of wood and push it through the sawdust so as to connect the inner and outer boxes: the ticking can now be heard plainly.

Exp. 22. Apply the ear to one end of a long rod of wood and note that the sound of a pin scratching the other end can be distinctly heard.

Exp. 23. Take two cylindrical wooden boxes like large pill boxes and join them by a long length of string, tape, catgut, or wire, having its ends securely connected externally to the bottoms of the boxes. It will be found that words spoken into one of the boxes can be clearly heard over considerable distances by applying the ear to the other box. This apparatus is sometimes called the drum telephone or string telephone. The sound waves originated inside one of the drum-like boxes travel along the string or wire to the other box.

It will be found from what follows that the propagation of sound presents all the characteristics of longitudinal wave motion in material media. Sound takes time to travel from point to point in a medium; it travels in each medium with a velocity determined by the properties of the medium, and it is subject to reflection, refraction, diffraction, and interference in accordance with the laws that apply to longitudinal wave motion.

70. Velocity of propagation of sound. It has been explained in Art. 29 that time is occupied in the propagation of wave motion and that the motion travels in any given medium with a definite constant velocity. Common observation shows that these statements also apply to the propagation of sound. In watching a cricket match the sound of a hit is always heard a little after the ball has left the bat; the report of a gun fired at a good distance from an observer is heard some seconds after the flash is seen; the explosion of a rocket shell is heard some time after the shell is seen to burst, and thunder is always heard a short time after the lightning flash is observed.

In all these cases the interval between the instant the production of the sound is *seen* and the instant the sound is *heard* is really the difference in the times taken by light and sound to travel from the source to the observer. The velocity of light is, however, so great that the time taken by it to travel over ordinary terrestrial distances is negligibly small, and the intervals referred to above are practically the times taken by the sounds to travel from the points of production to the observer. Careful observation will show that the magnitude of these intervals depends upon the distance of the source of sound from the observer; the greater the distance the greater the interval, and the shorter the distance the shorter the interval. Hence, if in any particular case the distance from the source of sound to the observer is known, and the interval of time between the instant of seeing the sound produced and the instant of hearing the sound is carefully measured, the average velocity of sound in air over the observed distance can be at once determined. For example, if a cannon at a distance of 13,420 feet from an observer is fired and the interval between the instant of observing the flash and the instant of hearing the sound is found to be exactly 11 seconds, then the average velocity of the sound in air over the observed distance is 13420/11 feet per second or 1120 feet per second. When observations of this kind are made for different distances it is found that under similar conditions the result obtained for the average velocity of sound in air is always the same, and, under ordinary conditions, comes to about 1120 feet per second. Experiments of this kind show that sound travels through air with a constant definite velocity, and it will be shown later, in Chapter XIII., that the magnitude of this velocity, as determined by experiment, is in accordance with the relation $V = \sqrt{E/D}$ (Art. 42), where E and D denote the elasticity and density of the medium. It is also found that the velocity varies with any change of conditions which affects the values of these constants. For example, the velocity varies with the temperature of the medium in accordance with the effect of change of temperature on the density of the medium (Art. 81). As the temperature rises the density decreases and the velocity increases; similarly, as the

temperature falls the density increases and the velocity decreases.

Similar experiments show that the velocity of sound in water is about 4700 feet per second, and this velocity is also found to be in close agreement with the value given by the theoretical relation $V = \sqrt{E/D}$.

These results show that sound travels through any given medium with a definite velocity, and that the magnitude of this velocity, as determined by experiment, is the same as that required by the theory that sound is propagated as longitudinal wave motion.

It is important to note that the velocity of sound is the same for all wave lengths. Sounds of high pitch travel in air at exactly the same rate as sounds of low pitch. This is conclusively proved by the fact that an observer listening to music played by a band at a distance hears the piece performed in correct time, just as it is played. If sounds of different pitch travelled at different rates it is obvious that the time of the piece would be disturbed and even the sequence of the notes might be altered.

The velocity of sound is also practically independent of the intensity of the sound. There is some experimental evidence to show that the velocity of sound decreases slightly, as the intensity diminishes, to a limiting value for sounds of very low intensity. This result, however, has not been satisfactorily confirmed. Sounds of very great intensity travel with slightly greater speed than ordinary sounds, but this result, if correct, applies only to certain exceptional cases.

71. Reflection of sound. Evidence derived from experiment and general observation proves that sound is subject to reflection in accordance with the laws that apply to the reflection of wave motion. The general principles involved in the reflection of wave motion have been considered in Chapter IX., and the detailed application of these principles in relation to Light is dealt with in the section on *Light*. The reflection of sound is not so easily studied experimentally as the reflection of light, but the same principles are involved and the same laws apply in each case.

Exp. 24. Arrange two tubes, each about 3 feet long and 2 inches in diameter, at an angle as shown in Fig. 57. Adjust a flat surface such as the surface of a book cover, a slate, or a square of wood or card-board, as shown at *a b*, so that the reflexion from this surface of the ticking of a watch at A is most clearly heard at B. A felt screen should be placed between A and B to cut off the direct sound from A. When this adjustment is made it will be found that the reflecting surface is equally inclined to both tubes, and that the axes of the two tubes and the normal to the surface are in the same plane. That is, the sound waves incident along the tube from A are reflected at the surface of *a b*, so that the angle of reflection is equal to the angle of incidence, and the directions of incidence and reflection are in the same plane as the normal to the surface.

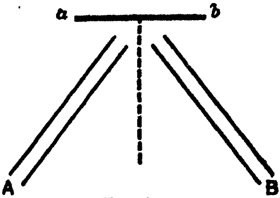


Fig. 75.

Exp. 25. Set up a large concave mirror, and place a watch on the axis of the mirror at a point between the focus and the centre of curvature of the mirror. Attach a length of rubber tubing to the stem of a small funnel, and, using the funnel as a sound collector, find the point on the axis of the mirror at which the reflection of the ticking of the watch is most clearly heard. This is the point at which the sound of the ticking is brought to a focus after reflection at the surface of the mirror, and it will be found that the relation between the distance of this point from the mirror and the distance of the watch from the mirror is in agreement with that required by the theory of the reflection of wave motion at a concave spherical surface.

This experiment may be modified so as to correspond to that usually given in illustration of the reflection of radiation. (*Heat*, **Exp. 120.**) The ticking of the watch at the focus of one mirror is reflected from this mirror to the second one, and there again reflected to the focus of the second mirror, where it can be distinctly heard by means of the funnel and tube collector.

In Chapter IX. it has been explained that the degree of smoothness necessary for regular reflection at any surface depends upon the wave length of the incident waves. The smaller the wave length the smoother must the reflecting surface be. A small amount of irregular reflection or diffusion, due to the action of surface in-equalities, takes place at all surfaces, but the loss of energy in the reflected wave on this account is comparatively small when the inequalities are small compared with the wave length.

The extent of surface necessary for regular reflection also depends on the wave length. The larger the wave length the larger must be the extent of the reflecting surface.

The wave length for audible sounds varies from about half an inch for sounds of high pitch to over thirty feet for sounds of low pitch, so that the degree of smoothness and the extent of surface necessary for the reflexion of sound depends upon the pitch of the sound. To reflect sounds of high pitch the reflecting surface must be fairly smooth, but need not be of large extent. To reflect sounds of low pitch, on the other hand, the surface may be comparatively rough, but it must be of sufficiently large extent. For example, the surface of a plank or plaster wall is smooth enough to reflect sounds of the highest audible pitch, and a comparatively rough and broken surface, such as the face of a cliff or mountain, is smooth enough and of sufficient extent for the reflexion of sounds of low pitch. An ordinary brick or stone wall where the surface inequalities are of the dimensions of about a centimetre will not reflect efficiently sounds of less than, say, half a metre wave length.

From what has been said in Art. **51** it will be understood that sound waves are reflected not only at the surface of a solid body, but also at the surface of separation between two different media or even at the surface of separation between two portions of the same medium, if the portions differ abruptly in density. For example, sound is reflected at the surface of water, at the surfaces of clouds, and at the surfaces separating currents of air of different temperatures.

Exp. 26. Arrange the apparatus of Exp. 24 as there described with a flat gas flame as the reflecting surface. It will be found that good reflection takes place at the surface of separation of the air from the hot gas of the flame. Reflection will also be found to take place at the surface of the hot current of air and gases ascending from the flame.

When regular reflection takes place at the surface of separation of two media the reflection is, in general, only partial; part of the incident wave is refracted into the second medium.

72. Echoes. The reflection of sounds from any suitably placed reflecting surface produces what is known as an *echo*. A cliff, a hill side, the walls of a building, the edge of a forest, clouds, all serve well to reflect sound, and it is by direct reflection at such surfaces that echoes are usually produced.

It has been found that when a sound is heard the sensation *persists* about one-tenth of a second after the exciting

stimulus has ceased. As a result of this it evidently follows that an echo of any sound can be fully and clearly heard only if the reflected wave returns to the ear of the observer one-tenth of a second *after* the direct wave left it. Hence, if the sound is of practically no duration, as in the case of a pistol shot or the blow of a hammer on an anvil, an echo will be heard distinctly if the sound takes one-tenth of a second to reach the reflecting surface and return to the observer. Under ordinary conditions the velocity of sound is about 1120 feet per second, so that it travels 112 feet in one-tenth of a second. The echo of a sound of very short duration can, therefore, be heard at a point distant about 56 feet from the reflecting surface. If, however, the exciting cause of a sound sensation lasts for one-tenth of a second the sensation itself will last for about one-fifth of a second, and a complete echo will be heard only if the distance of the reflecting surface from the observer is about 112 feet. With this distance between the observer and the reflecting surface the echo reaches the ear just as the first sensation ends. Similarly, if the direct wave acts on the ear for, say, ·4 second the sensation will last for about ·5 second, and a complete echo will be heard if the distance of the reflecting surface from the observer is about 280 feet. If, in this case, the distance were 112 feet instead of 280 feet the echo would reach the ear in about ·2 second, and would, therefore, for the next ·3 second for which the first sensation lasts be confused with, and indistinguishable from, this sensation. For the further ·1 second for which the echo lasts it would be clearly heard for about ·2 second as an echo of the last quarter of the initial sound.

In the case of words uttered by the human voice it is found that the most rapid rate at which short syllables can be uttered and *heard as distinct sounds* is about five per second. That is, the sensation for any short, sharply uttered syllable lasts for about one-fifth of a second, and a complete echo of it can, therefore, be heard only if the distance of the reflecting surface is at least 112 feet. If a number of syllables are rapidly spoken then, unless the reflecting surface is at a sufficiently great distance, the echo of the first syllables will be confused with the last

directly spoken syllables, and only the echo of these last syllables will be clearly heard. Thus, if the distance of the reflecting surface from the observer is about 112 feet, then only the echo of the last syllable uttered will be clearly heard, for the echo of this syllable will reach the ear just as the sound of the syllable itself ceases to be heard. If the distance is 224 feet, then, evidently, the echo of the last two syllables can be heard; and if the distance is n times 112 feet, then the echo of the last n syllables can be heard. Echoes which repeat two or more of the syllables uttered are sometimes called *polysyllabic echoes.*

The data of these calculations, the period of persistence of a sound sensation, and the number of distinct syllables which can be uttered and heard in a second, are, of course, only approximately known and vary within narrow limits for different individuals. It must be remembered, too, that the distance of 112 feet given above in connection with polysyllabic echoes assumes that the conditions are such that the velocity of sound is 1120 feet per second. If the velocity were 1100 feet per second, then 110 feet would be distance travelled in one-tenth of a second.

When a sound is reflected from a number of suitably placed reflecting surfaces a number of echoes may be heard, and if the surfaces are at different distances the succession of echoes may last for some time. The rumbling and rolling of thunder, for example, is probably the echoing of the peal of thunder from a number of reflecting surfaces such as surfaces of separation between atmospheric currents, cloud surfaces, rocks, mountain sides, and cliffs. With two suitably placed surfaces, too, it is possible for a sound to suffer successive reflexion from one surface to another, and a very prolonged echo may thereby be produced.

73. Musical notes produced by reflection. If a short, sharp sound, such as may be made by striking two stones together, is subjected to repeated reflection from the walls of a room or passage, or from a flight of stairs, the successive echoes may reach the ear in sufficiently rapid and regular succession to produce the sensation of musical note. The

musical ring which sometimes accompanies the sound of a footstep in a bare hall or corridor is an instance of this effect.

Similarly, if a short, sharp sound is produced near a paling or railing the sound may, if its wave disturbance is sufficiently short, be reflected from each individual railing, and the successive reflections will reach an observer in a regular succession which may be rapid enough to produce the sensation of a musical note. This explains the musical sound which is heard when a sound, such as may be produced by striking two keys together, is produced near an iron railing.

74. Other illustrations of reflection of sound. If a room is of such a size and so constructed that the echoes from the walls interfere with and obscure words and sounds almost as soon as they are produced, the room is said to be bad acoustically for speaking or singing. To remedy this defect curtains and screens are sometimes put up to prevent the reflection which gives rise to the echoes.

The action of a speaking tube is a good illustration of the reflection of sound. The wave energy which, without the action of the tube, would spread out into a rapidly increasing volume of air is directed along the tube and, by repeated reflection from the inner surface, confined to the air in the tube. For this reason the intensity of the sound does not decrease according to the law of inverse squares, as it should do if the disturbance spread out radially from the point of production, but remains practically constant, subject only to loss by the dissipation of energy which accompanies reflection.

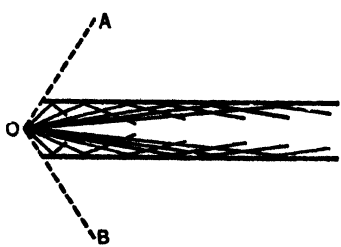


Fig. 58.

From Fig. 58 it can be seen that the wave which would, in the absence of the tube, spread out radially from O within the solid angle represented by A O B is confined by reflection from the

wall of the tube to the air within the tube, along which it travels with very little diminution of intensity with distance.

The ear trumpets used by deaf persons also furnish illustration of the reflection of sound. The wave energy which enters the wide end is directed by repeated reflexions through the trumpet to the ear, and as the volume of the air by which the energy is carried decreases as the ear is approached, the intensity of the wave motion is increased and the sound is more easily heard.

Another illustration of the reflection of sound is found in the case of a *whispering gallery*. In the circular gallery in the dome of St. Paul's Cathedral a whisper is reflected round the wall of the gallery so as to be audible at any point of its circumference. The action of the gallery is practically that of a circular speaking tube. If sound is uttered at O (Fig. 59) the wave radiates in all directions from O, but the portion included between any two lines O A and O B is evidently reflected round and round the gallery in the air between the wall and the cylindrical surface B C, to which the lines O A and O B are tangential.

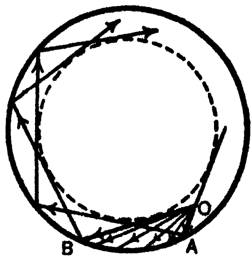


Fig. 59.

In this way a whisper at O can be heard anywhere near the wall of the gallery.

75. Refraction of sound. When sound passes from one medium into another, in which the velocity of transmission is different, it is found to be subject to refraction in accordance with the laws of refraction of wave motion as explained in Art. **54**.

Sound, like light, should therefore be refracted by lenses and prisms of suitable material, and the geometrical relations which apply in the refraction of light at plane and spherical surfaces apply also for sound.

A lens for the refraction of sound should be of gas, and not of any dense substance, otherwise most of the sound is

reflected at the first surface. A convenient sound lens is a collodion balloon filled with carbon-dioxide, in which sound travels more slowly than in air. When a watch is hung up behind the balloon the ticking may be heard very distinctly at a certain point in front. The position of this point is found by trial to correspond to the calculated focus of the waves which pass through the lens. A better lens is obtained by cutting out two convex circular sheets of collodion and gumming their edges to the opposite sides of a metal hoop.

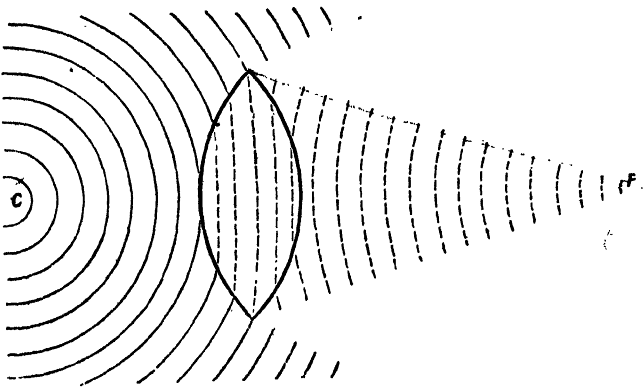


Fig. 60.

The diagram (Fig. 60) shows a section of the lens and the refraction of a spherical wave through it. As the thickness of the lens increases from the edge to the centre the wave is retarded most at the centre, so that the wave front of the emergent wave is concave and converges to a focus at F.

The refraction of sound has no important applications. It is of interest mainly as experimental evidence in favour of the theory that sound is propagated as longitudinal wave motion in the material media through which it

It should be noted that, although sound is refracted in accordance with the laws that apply to the refraction of light, dispersion, as in the case of light, cannot take place with sound waves. This is due to the fact that the velocity of sound in any medium is the same for all wave lengths. A compound sound made up of a number of simple sounds cannot therefore be analysed by dispersion in the same way as white light is analysed by dispersion in a prism into the simple colour constituents which make up the prismatic spectrum.

76. Interference of sound : beats. In Art. 56 it has been shown that *interference* is one of the phenomena of wave motion. Experiment shows that sounds may be made to exhibit interference effects, and the theory that sound is wave motion is thereby confirmed.

The most familiar illustration of sound interference is afforded by the *beats* heard when two sounds of *nearly* the same pitch are heard at the same time. Thus, if two tuning forks of nearly the same frequency are sounded together, the sound heard possesses a very characteristic and well-marked throbbing or beating effect. Thus, if the frequencies of the forks are, say, 256 and 257 per second respectively, the combined sound swells out and dies away rhythmically once in each second. This *beating* effect is due to interference. If the two forks make respectively 256 and 257 vibrations per second, it is evident that if at any instant they are in the same phase, then *half a second later* they will be in opposite phases, and a whole second later they will again be in the same phase. It follows from this that, if at any instant the two trains of waves at the point where they reach the observer's ear are in the same phase, then half a second later they will be in opposite phases, and a whole second later they will again be in the same phase. When the waves are in the same phase the "interference" is helpful and the intensity of the sound is increased; when the waves are in opposite phases the "interference" is *destructive* and the intensity of the sound is decreased and may become zero. The result of these interference effects evidently is that the

resultant sound heard by the observer is marked by a regular sequence of maxima and minima of loudness, with an interval of a second between two consecutive maxima or minima. The maxima occur when the waves from the two forks are in the same phase and the minima when the waves are in opposite phases.

A single beat may be considered to occupy the interval between two consecutive maxima or minima. As the swell of the sound from its minimum to its maximum intensity, followed by the fall from this maximum to the minimum again, is the most noticeable feature of beating, a beat is usually taken to extend from minimum to minimum.

It will be clear on consideration that when any two sounds of nearly the same pitch are heard together, the number of beats heard per second must be equal to the difference of the frequencies of the two sources. Thus, if the frequencies are 254 and 258 respectively, the sources must evidently be in the same phase *four* times in each second, and a single beat must therefore occupy a period of one-quarter of a second. That is, when the difference of the frequencies is four per second there are four beats per second.

When the number of beats per second is more than ten, it is very difficult to separate them, but their presence may cause discord. When the number per second exceeds thirty, they cease to be distinguishable, and may have no disagreeable effect.

77. Effect of wind on the propagation of sound. When sound is travelling in wind the velocity with which it travels in any direction is evidently the resultant of the velocity of sound in still air and the velocity of the wind. That is, when sound is travelling in the same direction as the wind its velocity is the sum of the two velocities, and when travelling in the opposite direction its velocity is the difference of the two velocities.

Under general conditions for a steady wind the velocity of the air is least at the surface of the earth and increases as the height above the ground increases. Hence, in general, when sound travels with the wind its velocity is

least near the surface of the earth and increases as the height about the earth's surface increases. Similarly, when sound travels against the wind its velocity is greatest (because least retarded) near the surface and increases upwards from the surface.

If, therefore, A B (Fig. 61) represent the wave front of a sound wave, it is evident that if it advances with the wind it will be refracted, after a short interval, into the

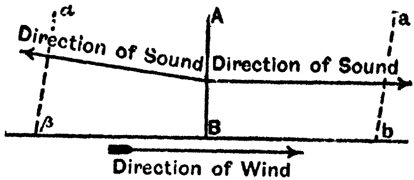


Fig. 61.

position $a\,b$, and the energy of the wave is directed downwards, out of the upper air, towards the surface of the earth. This explains how sounds are, as it is said, "carried" by the wind. On the other hand, if the wave front advances against the wind it will be *refracted* after a short interval into the position shown at $\alpha\,\beta$ in the figure, and the energy of the wave is directed upwards into the upper air away from the surface of the earth. For this reason sounds are not heard at the surface of the earth at any great distance to the leeward of the source of sound. This effect of wind is sometimes called *wind refraction.*

78. Effects of the distribution of temperature in the air. Refraction of an exactly similar character and attended by the same effects is produced by the variation of the temperature of the air with height above the earth's surface. Generally, in the day-time the temperature decreases as the height increases, and the velocity of sound is therefore greatest near the surface of the earth, and decreases as height increases. Under these conditions, then, sound is refracted in the same way as when it travels against wind.

That is, for observers at the earth's surface, sounds do not travel far when the fall of temperature with increase of height is steady and well marked, as on a calm hot day. If the rate of fall of temperature upward becomes too great, the statical equilibrium of the air is upset and convection currents are set up. Under these conditions the sound waves are subject to so many reflections and refractions at the surfaces of separation of the various currents that they are quickly dispersed and broken up, and sounds are said to "carry" badly through the air.

Under certain conditions the air may be cooler near the earth's surface than at a higher level, and for a certain height there may be a rise of temperature with height. For example, in the evening after a very hot day the air near the earth cools more rapidly than higher up, and for a time a layer of air exists in which the temperature increases with height above the earth's surface. Under these conditions sound is refracted in the same way ~~and~~ as when it travels with the wind, and sounds are found to carry to great distances along the ground.

When the conditions are such that the air is very still and the temperature is practically uniform for a considerable height, the sound waves travel through a perfectly homogeneous medium, and as there is very little dissipation of energy by reflection or refraction, sounds travel well, and may be audible at great distances.

79. Conditions favourable to the propagation of sound. *The* condition essential for good propagation of sound waves is the *homogeneity* of the medium. If the medium is at rest, free from internal motion such as currents or eddies, and uniform throughout in temperature, density, and structure, the conditions are those most favourable for the propagation of sound waves in the medium. In the case of a medium containing small particles, the particles will not interfere with the propagation of wave motion through the medium, so long as the dimensions of the particles are small compared with the wave length. Thus, although smoke particles in mid air make it more or less opaque to light waves, they do not interfere in the least with the

propagation of sound waves through the air. In the same way sound is readily transmitted through any screen or curtain which does not interfere with the continuity of the medium in which it is placed. For example, a screen of silk or loosely woven cloth interferes little with the propagation of sound in air, but a single sheet of *wet* silk wholly prevents transmission and acts as a reflecting surface.

It is found that foggy air is specially good for the propagation of sound. This is explained by the fact that fog can only form in homogeneous air, and foggy air is therefore specially homogeneous air containing fog particles which are much too small in dimensions to interfere in the slightest degree with the propagation of ordinary sound waves.

At the request of the Trinity Board, Tyndall conducted a long series of observations off the South Foreland with the object of determining the distance to which various sound signals could penetrate under all sorts of weather conditions. These observations proved that there is no relation between the visual clearness of the atmosphere and its transparency to sound. On some beautifully clear days sounds were extinguished at a distance of less than three miles, while on other occasions of much less optical transparency they penetrated to a distance of nearly thirteen miles. The presence of fog, falling snow, rain, or hail had no sensible effect in obstructing sound and often coincided with atmospheric conditions specially favourable to the transmission of sound. Tyndall concluded that the stifling of sound in air was due almost entirely to the scattering of the sound waves by repeated reflection and refraction at the surfaces of separation of air currents or of masses of air at different temperature and in different hygrometric states. These masses of air Tyndall called *acoustic clouds*, and he attributed *aërial echoes* which he observed to reflection at the surface of these clouds.

EXERCISES XII.

1. Describe an experiment for showing that sound is capable of being transmitted through liquids.

2. How would you show that sound is not transmitted by a vacuum?

3. What is the velocity of sound in air? On what grounds can it be asserted that musical tones of high and low pitch travel at the same rate?

4. How may the distance of a lightning flash be approximately calculated?

A thunder clap was heard 10 secs. after the lightning flash was seen. How far off was the explosion? (The velocity of sound = 1120 ft. per sec.)

5. Explain how the condensations and rarefactions constituting a wave of sound are produced. How is a wave propagated through a gas?

6. A bell when struck emits a note of a certain frequency. Is the wave-length in air corresponding to this note of the same length on a warm day as on a cold day? Give reasons for your answer.

7. If a pistol-shot is fired at one end of a long tube filled with air two reports are heard at the other end. Explain this.

8. What experiments would you perform to illustrate the fact that sound can be reflected from hot air?

9. Explain any method by which the ticking of a watch may be made audible to a person at the other end of a large room.

10. Explain how a speaking tube acts.

11. A person is walking between two parallel walls which are near together, and hears a prolonged echo of each footstep. Explain how the echo is produced.

12. Describe an experiment illustrating the use of sound-boards in making the vibrations of a wire audible.

13. A cannon is placed 550 yards from a long perpendicular line of smooth cliffs. An observer at the same distance from the cliffs hears the cannon-shot four seconds after he sees the flash. If the velocity of sound is 1100 ft. per second, when will he hear the echo from the cliffs?

14. An echo repeated six syllables. The velocity of sound is 1120 ft. per sec. What was the distance of the reflecting surface?

15. Describe an experiment to show that sound may be refracted. Can sound be dispersed?

16. What would happen if two sound waves exactly alike were to meet one another in the air, moving in opposite directions?

17. Two notes are sounding and 10 beats per second are heard. If the lower note has a frequency of 530 what is the frequency of the higher note?

18. A strong wind is blowing. When I am some distance on the windward side from a sounding whistle I can easily hear it. When I am the same distance away on the leeward side I find difficulty in hearing it. Why is this?

19. Explain why in the evening after a hot day sounds "carry" very well along the surface of the ground.

20. Why is foggy weather sometimes specially good for the propagation of sound?

21. How do you account for the fact that the distance at which a loud sound (such as the discharge of a cannon) is heard varies considerably from day to day?

EXAMINATION QUESTIONS.

1. A stone is dropped perpendicularly into still water and produces a series of concentric waves. At the end of 5 seconds there are 50 concentric troughs and crests and the boundary of the outermost is a circle of 1·5 metres radius. Find the wave-length and velocity of propagation of the disturbance. Find also the period of oscillation of a water-particle.

2. Explain how to set up an experiment and describe carefully the observations you would make, in order to determine the laws of reflection of sound.

3. Explain how an echo is produced. A man cracks a whip near a line of vertical palings, and after each crack a shrill note is heard. Explain this.

CHAPTER XIII.

THE VELOCITY OF SOUND.

80. Calculation of the velocity of sound in air. In Chapter VII. it has been shown that the velocity of longitudinal waves, that is, sound waves, in any fluid medium is given by the relation $V = \sqrt{E/D}$, where E denotes the modulus of bulk elasticity and D the density of the medium.

The *velocity of sound in air* is readily calculated from this relation. As explained in Art. **43**, the elasticity involved in the propagation of sound waves in a gas is the adiabatic elasticity, and the modulus of volume elasticity for adiabatic strains in a gas is shown to be given by γ P, where P denotes the pressure of the gas, and γ is the ratio of the two specific heats of the gas.

The velocity of sound in a gas is therefore given by the relation $V = \sqrt{\gamma P/D}$, where P expresses the pressure of the gas and D its density under existing conditions.

For the purpose of this formula P and D must evidently be expressed in appropriate and consistent units; P must be given in units of force per unit of area, and D in units of mass per unit volume. Thus, in the C.G.S. system, P would be expressed in dynes per square cm., and D in grammes per c.c., and in these units the value of V would be given in cms. per second. Similarly, in the English F.P.S. system, P should be in poundals per square foot, D in pounds per cubic foot, and V in feet per second. The pressure of a gas is usually specified in terms of the height of the barometric column, so that in applying the above

formula it is necessary to convert the pressure specified in this way into proper units. Thus the pressure indicated by a barometric height of 760 mms. (corrected to 0° C.) is appropriately expressed as $76dg$ dynes per square cm., where d represents the density of mercury at 0° C. and g the acceleration due to gravity at the place of observation. The value of d is generally taken as 13·596 grammes per c.c., and g may be taken, with sufficient accuracy for most places in England, as 981 cms. per second per second.

The value of γ, the ratio of the two specific heats of a gas, depends upon the number of atoms in the molecule of the gas. In the general normal case of diatomic molecules the value of γ is about 1·41; in the exceptional cases of monatomic and triatomic molecules the values are 1·66 and 1·26 respectively.

Hence, in air at normal temperature and pressure P may be taken as $76 \times 13{\cdot}596 \times 981$ dynes per square cm., or $1{\cdot}0136 \times 10^6$ dynes per square cm., and

$$E = 1{\cdot}41\,P = 1{\cdot}41 \times 1{\cdot}0136 \times 10^6 \text{ dynes per square cm.}$$

Also D, for *dry* air, is ·001293 grammes per c.c., and V is therefore given by the relation

$$V = \sqrt{\frac{1{\cdot}41 \times 1{\cdot}0136 \times 10^6}{{\cdot}001293}} \text{ cms. per sec.} = 33{,}240 \text{ cms. per sec.}$$

That is, the velocity of sound in *dry* air at normal temperature and pressure is about 332 metres per second, or about 1090 ft. per second. This result agrees with that obtained by experiment, as detailed in Art. **84**.

The velocity of sound in hydrogen, or any other gas, may be calculated in the same way. Thus, for hydrogen at normal temperature and pressure

$$E = 1{\cdot}41 \times 1{\cdot}0136 \times 10^6, \quad D = {\cdot}00009 \text{ gramme per c.c.,}$$

and V is given by

$$V = \sqrt{1{\cdot}41 \times 1{\cdot}0136 \times 10^6/{\cdot}00009} \text{ cms. per second}$$
$$= 126{,}200 \text{ cms. per second.}$$

It evidently follows from the relation $V = \sqrt{\gamma\,P/D}$ that, under the same conditions, the velocities of sound in any two gases for which the value of γ is the same are

inversely proportional to the square roots of their densities. For in one gas we have $V_1 = \sqrt{\gamma P/D_1}$, and in the other $V_2 = \sqrt{\gamma P/D_2}$, and therefore $V_1/V_2 = \sqrt{D_2/D_1}$. For example, in the case of air and hydrogen at normal temperature and pressure, from the data given above, we have $V_1/V_2 = \sqrt{90/1293}$, and from this relation either V_1 or V_2 may be calculated if the other is known.

81. Influence of pressure, temperature, and hygrometric state on the velocity of sound in air. The velocity of sound in a gas, as determined by the relation $V = \sqrt{E/D}$, is evidently subject to correction for variation in pressure, temperature and humidity. In the case of a gas which obeys Boyle's law, the formula $V = \sqrt{E/D}$ reduces to $V = \sqrt{\gamma P/D}$, and as the density of the gas at constant temperature varies directly with the pressure, and the ratio P/D is constant for all values of P, the value of V for a given temperature is therefore unaffected by change of pressure. That is, variation in pressure only has no effect on the value of V and, at any given temperature, the velocity of sound in a gas is the same at all pressures.

Variation in temperature, however, causes change in the density of the gas and therefore affects the velocity of sound in the gas. A rise of temperature causes the density, D, to decrease and therefore increases the value of V; similarly, a fall of temperature increases D and decreases the value of V. That is, the velocity of sound in a gas increases with rise of temperature.

The quantitative nature of the effect of the variation of temperature on the velocity of sound in a gas is readily stated. If D_o and D_t denote the density of the gas at $0°$ C. and $t°$ C., then it is known* that $D_t = D_o/(1 + a\,t)$, where a denotes the coefficient of cubical expansion of the gas. Hence, in applying the relation $V = \sqrt{\gamma P/D}$ for a gas *at* $t°$ *C.* we must write $V^t = \sqrt{\gamma P/D_t}$, that is, $V_t = \sqrt{(\gamma P)/(D_o/1 + a\,t)}$, or $V_t = \sqrt{(\gamma P/D_o)\ (1 + a\,t)}$. But $V_o = \sqrt{\gamma P/D_o}$, and therefore $V_t = V_o\sqrt{1 + a\,t}$. The

* *New Matriculation Heat*, Art. 40.

value of a for a gas is ·00366, so that $V_t = V_0 \sqrt{1 + \cdot 00366\, t}$. If we take the value of V_0 as 33240 cms. per second, then $V_t = 33240 \sqrt{1 + \cdot 00366\, t}$, or $V_t = 33240 + 61\, t$. Similarly, if we take $V_0 = 1090$ ft. per second, then $V_t = 1090 \sqrt{1 + \cdot 00366\, t}$ or $V_t = 1090 + 2\, t$. That is, the velocity of sound in a gas increases with rise of temperature by about 61 cms. per second, or about 2 ft. per second for each degree (Centigrade) rise in temperature.

The humidity or presence of water vapour in a gas affects the velocity of sound by altering the density. The density of water vapour relative to air at the same temperature and pressure is about ·62, so that in the case of air the presence of water vapour reduces the density, and the velocity of sound in moist air is therefore greater than in dry air.

From what has been said it is evident that in applying the formula $V = \sqrt{\gamma P/D}$ to determine the velocity of sound in a gas, it is only necessary in order to correct for variation of pressure, temperature, and humidity, to determine the correct value of D under the existing conditions, and to calculate the value of V given by substituting this value of D, and the specified value of P, in the formula.

Example. Find the velocity of sound in air at 15° C., given that the dew-point is 10° C. and the barometric height (reduced to 0° C.) is 759·13 mm.

Under these conditions the density of the moist air is found to be 1·219 grammes per litre,* that is $D = \cdot 00129$ in grammes per c.c. The value of P is the pressure due to 759·13 mm. of mercury expressed in dynes per square cm.; that is, $1\cdot 0125 \times 10^6$ dynes per square cm. From these data the value of V given by the relation $V = \sqrt{\gamma P/D}$, is $V = \sqrt{1\cdot 41 \times 1\cdot 0125 \times 10^6 / \cdot 001219}$ cms. per second, or $V = 34{,}200$ cms. per second.

The effect of aqueous vapour in the air is not fully corrected for by correcting the density only; moist air does not strictly obey Boyle's law, so that its elasticity is only approximately measured by its pressure. Also the ratio of the two specific heats for the mixture of air and water vapour is not exactly 1·41. Under ordinary atmospheric conditions, however, these sources of error in applying the formula $V = \sqrt{\gamma P/D}$ as above, to determine the velocity of sound in air under given conditions, may be neglected.

* See *New Matriculation Heat*, Art. 134.

82. Laplace's correction of Newton's calculation of the velocity of sound in air. The calculation of the velocity of sound in air from the relation $V = \sqrt{E/D}$ was first made by Newton. In making the calculation, however, he assumed that the compressions and rarefactions of the waves were effected under *isothermal conditions*, and on this assumption deduced from Boyle's law, as in Art. **11**, that the modulus of elasticity applicable to the case was measured by the pressure of the gas. Newton, therefore, stated that $V = \sqrt{P/D}$, where P denotes the pressure and D the density of the air. With data then available for air this formula gave the value of V in air about 979 ft. per second.

This value Newton recognised to be much below the value obtained by experiment, and he endeavoured to explain the difference in the results by assuming that sound travelled instantaneously through the air molecules, and that the 979 feet obtained by the formula as the distance travelled in one second was really only the sum of the distances between the molecules and not the total distance. He also assumed that the water vapour present in the air took no part in the transmission of the waves. These assumptions are, however, now known to be inadmissible, but it was only in 1816 that the true explanation of the mistake made by Newton was pointed out by **Laplace**. Laplace showed that the very rapid compressions and rarefactions which take place in sound waves were not effected under isothermal but under adiabatic conditions, and that the value of E was not given by P, but by γP as already explained.

83. Calculation of the velocity of sound in water. The velocity of sound in any liquid medium for which E and D are accurately known may be calculated from the formula $V = \sqrt{E/D}$, and the result obtained usually agrees satisfactorily with the result obtained by experiment. Thus in the case of water at 15° C. the adiabatic value of E is $2{\cdot}23 \times 10^{10}$ dynes per square cm., and the value of D

is approximately 1 gramme per c.c., so that the value of V from the formula is given by

$$V = \frac{\sqrt{2 \cdot 23 \times 10^{10}}}{1} \text{ cms. per second}$$
$$= 150{,}000 \text{ cms. per second.}$$

The experimental determinations of the velocity of sound in water give values between 1400 and 1500 in metres per second.

84. Experimental determination of the velocity of sound in air. It has already been explained in Art. 70 how the velocity of sound in air may be determined by observation of the time taken by a sound, such as the report of a gun, in travelling over a known distance. It has also been indicated in Art. 72 how this velocity may be found by noting the interval of time which elapses between the utterance of a sound and the return of its echo from a reflecting surface at a known distance from the observer. For example, if the echo of a sound returns to an observer from a reflecting surface at a distance of 843 feet in 1·5 seconds after it starts out from the point of observation, then the sound obviously travels over 1686 feet in 1·5 seconds, and its average velocity over this distance is therefore 1124 feet per second.

From the beginning of the eighteenth century many determinations of the velocity of sound in air have been made by means of this direct method of experiment. The time taken by the report of a cannon in travelling between two stations a known distance apart was measured, and the velocity calculated from the data so obtained. In the earlier experiments the effect of wind and the influence of the temperature and hygrometric state of the air on the result were more or less neglected. The methods of measuring the time interval were also less accurate than those adopted in later experiments.

The effect of wind was first determined experimentally by Derham's experiments. In 1708 Derham found, after repeated experiments, that the report of a cannon travelled over a distance of 12½ miles, between Blackheath and the

tower of Upminster Church in Essex, in a time which varied between 55·5 and 63 seconds according as the strength and the direction of the wind varied. The shorter interval was obtained when the directions of the wind and sound were the same, the longer interval when these directions were opposite. As the result of this experiment Derham gave the velocity of sound in air as 1142 feet per second, but he made no record of the temperature or hygrometric state of the air, as he considered they had no influence on the result.

In later experiments of this type the effect of wind has usually been eliminated by the method of *reciprocal observation*. A gun is fired at *each* of the two stations of observation and the times taken by the report in travelling over the distance between the two stations in each direction are carefully measured. By taking the mean of these two times as the correct time of travelling over the distance the effect of the wind is practically eliminated, provided its velocity is constant during the time of the observations and not unusually great. The temperature and humidity of the air were also, in most cases, determined as far as possible, and the proper corrections applied to the results of the experiments. In the determination of the time interval taken by the sound in travelling over the measured distance special methods and special mechanical and electrical devices were from time to time adopted to secure the greatest possible accuracy in the determination.

The general result of the more reliable experiments gives the velocity of sound in dry air at 0° C. to be about 332 metres per second or 1090 feet per second. This agrees well with the result required by the formula $V = \sqrt{E/D}$.

A series of very careful determinations were made by Mr. Stone, of Cape Town Observatory, in 1871. The one o'clock gun fired at Port Elizabeth was taken as the source of sound. Two observers were stationed on the line joining the gun to the Observatory and the time taken by the sound in travelling over the distance between the observers was carefully determined.

From these experiments Stone gives the velocity of sound in dry air at 0° C. as 1090·6 feet per second.

85. Determinations of special interest. Some of the experimental determinations of the velocity of sound in air were made under special conditions which give them special interest.

In 1845 two observers, Bravais and Martin, experimented between two stations on the Faulhorn *at levels differing in height* by about 2000 metres. The result of their experiments gave the velocity of sound in air at 0° C. as 332·4 metres per second. This result confirms the theoretical deduction that the velocity of sound in a gas obeying Boyle's law is independent of its pressure.

In 1889 Greely made a number of determinations of the velocity of sound in air in the Arctic regions at temperatures between − 10° C. and − 45° C.

As the result of his determinations he found that the velocity of sound in dry air at any temperature $t°$ C. is given in metres per second by the relation

$$V = 333 + \cdot 6\, t.$$

This result is in good agreement with the theoretical result obtained in Art. **81**, and confirms the theory on which the temperature correction is made, by direct experiment.

In 1864 Regnault made a very careful determination of the velocity of sound in the open air, and the results he obtained as the mean of a very large number of experiments over two distances of 1280 metres and 2445 metres gave the velocity of sound in dry air at 0° C. as 331·37 metres per second over the shorter distance and 330·7 metres per second over the longer distance. These results seem to show that the velocity of sound in air decreases as the intensity decreases, tending probably to a lower limit as the intensity diminishes. There is some experimental evidence in support of this conclusion, but no entirely satisfactory confirmation has yet been obtained.

Regnault also made a large number of determinations of the velocity of sound in pipes of diameters varying from a decimetre to a metre. He found as the general results of his experiments that in pipes or tubes the velocity of sound in air certainly decreases as the intensity diminishes, and also that the velocity increases as the

diameter of the tube increases. For example, in a tube ·108 metre in diameter the velocity was found to be 324·25 metres per second, while in a tube 1·1 metre in diameter it was 330·3 metres per second.

86. Experimental determination of the velocity of sound in water. In 1826 Colladon and Sturm determined the velocity of sound in water by direct experiment in the Lake of Geneva. A bell placed at a depth of about a metre in water was used as the source of sound. The arrangement for striking the bell (Fig. 62) was such that at the instant the stroke was made a charge of powder, *p*, was fired by a lighted fuse, *m*, so that the observer at the receiving station could mark the instant of striking by means of the flash of the discharge. The sound of the bell stroke travelled through the water to the observer at the distant receiving station, where it was heard by means of a long ear trumpet fixed with its receiving end in the water. It was found during the experiments that the bell could be heard right across the lake, a distance of nearly ten miles. By these means it was possible to determine with considerable accuracy the time taken by sound to travel over a measured distance through water, and from data so obtained the velocity of sound in water was calculated. Colladon and Sturm gave 1435 metres per second, or about 4700 feet per second, as the mean result of their experiments.

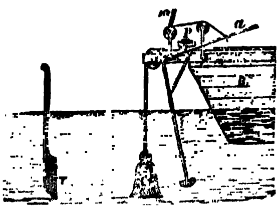


Fig. 62.

87. Indirect methods of determination. Indirect experimental methods of determining the velocity of sound in air and other media by measurements connected with the vibration of solid rods or fluid columns are dealt with in Art. 115.

It may, however, be here noted that these methods are, in general, direct applications of the relation $V = n\lambda$ explained in Art. 36.

It is obvious that if the wave length λ for a note of known frequency n can be determined experimentally for any medium, then the velocity of sound *in that medium* is given by $V = n\lambda$. For example, if a note of frequency 300 has a wave length of 34 feet in wood as determined by the period of vibration of a wooden rod of known length, then the velocity of sound along the rod is 300×34 feet per second or 10,200 feet per second.

EXERCISES XIII.

1. Newton showed that the velocity of sound in a gas $= \sqrt{\frac{P}{D}}$. What was his error? Explain Laplace's correction.

2. Show that sound travels faster in moist air than in dry air under the same conditions of temperature and pressure.

3. Describe any careful experiment made to find the velocity of sound in air.

4. Explain how the velocity of sound in water has been experimentally determined.

5. What effect does a rise in temperature produce in the velocity of sound? When the temperature is at the freezing point a sound passes from *A* to *B* in 10 seconds. Find the temperature if the sound could pass in 9·652 seconds.

6. A tube 1000 ft. long is filled with oxygen. Find how quickly a sound will travel from one end to the other, it being given that the density of oxygen is 16 times as great as that of hydrogen, and that the velocity of sound in hydrogen is 4200 ft. per second.

7. State how the velocity of sound depends upon the pressure, density, and temperature of the air. Find the velocity of sound in ft. per sec. at 27° C.

8. Calculate approximately the speed of sound in ft. per sec. in air at the following temperatures Centigrade: 20°, 15°, 30°.

9. Find approximately the temperature of the air when the speed of sound is 1000, 1120, 1200, and 980 feet per sec.

10. Calculate the distance of a lightning discharge when between the flash and the first sound of the thunder the following intervals of time elapsed—½ sec., 4 secs., 2½ secs.: the temperature in the first two cases being 10° C., and in the third 18° C.

11. Find at what temperature the velocity of sound in air is double the velocity of sound in air at the temperature of the freezing point of water.

12. Taking 1120 ft. per second as the speed of sound in air, find the number of vibrations which a C fork of frequency 264 must make before the sound is heard at a distance of 154 ft.

13. What is meant by a wave of sound, and by the length of a wave? Waves of sound, the frequency of which is 256, pass from a stratum of hot air to a layer of cold air. In the cold air the speed is 1120 ft. and in the hot air 1132 ft. Find the wave-length in each case.

14. A certain tuning-fork makes 256 vibrations per sec. Calculate the length of the wave it produces when the temperature of the air is 15° C.

15. If the same fork were sounded in hydrogen at the freezing point, how long a wave would it produce?

16. One wire of a piano is known to produce a wave 4 ft. long when the temperature of the air is 12° C. How many vibrations per sec. does the wire make?

17. A tuning-fork making 300 vibrations per sec. produces, when sounded in a certain gas, a wave 3 ft. long. Calculate the speed of sound in the gas.

18. The same fork sounded in air gives a wave 3·7 ft. long. Calculate the temperature of the air.

19. An echoing cliff at a distance of 650 ft. repeats six syllables. What is the temperature?

20. I speak before a reflecting surface 500 ft. away. The temperature is 15° C. How many syllables will be echoed?

CHAPTER XIV.

PITCH.

88. Experimental determination of pitch. The experimental determination of the pitch of a musical note is effected by determining the frequency of vibration of the source of the note. This determination may be made in three general ways: (1) by direct determination of the frequency of vibration of the source by suitable graphic or stroboscopic methods; (2) by adjusting the pitch of a note produced by a source of known and adjustable frequency (such as the cardboard syren described in Art. **63**) in unison with the note whose pitch is to be determined; and (3) by comparing the pitch to be determined with that of a suitable standard source of constant known frequency, such as a standard tuning fork. The methods (2) and (3) are essentially the same and differ only in the character of the source employed as a standard. In both methods the adjustment or comparison of pitch is usually made by means of *beats*. When the two notes compared are nearly of the same pitch, beats can generally be distinctly heard, and the number of beats per second gives, as explained in Art. **76**, the difference of the frequencies of the two sources, so that if the frequency of one is known the frequency of the other can be determined without further adjustment.

89. Direct determination of frequency. As already indicated, direct determinations of frequency are made either by a graphic method, in which the vibrating source records its frequency by writing a trace of its motion on a suitably arranged moving surface on which definite time intervals are also recorded, or by a stroboscopic method, in which, by means of intermittent illumination or observation, the motion of the source can be accurately compared with that of a suitable standard.

The commonest graphic method is that by which the frequency of a tuning-fork is determined by arranging that a fine style attached to one prong of the fork traces the motion of the prong on the smoked surface of a revolving cylindrical drum. The surface of the drum is generally formed of a sheet of smoked paper which can be removed when necessary for the examination of the trace of motion. In order to record definite time intervals on the trace, the fork and the drum are usually connected to the secondary terminals of an induction coil in which the current in the primary coil is periodically interrupted by the action of the pendulum of a standard clock. At each interruption of the primary current a spark passes between the style and the surface of the drum and leaves a little mark on the trace. The drum may be rotated by hand or by clock-work; it is evidently not necessary that its motion should be uniform, for the time intervals determined by the clock pendulum are indicated on the trace by the spark marks. If, however, the motion is irregular the trace will also be irregular in character, being unduly drawn out at some points and unduly compressed at others. The speed of rotation evidently requires to be adapted to the frequency of vibration in order to give a satisfactory trace.

A common form of this arrangement is shown in Fig. 63, and Fig. 64 shows the kind of trace given by its use, with a spark mark at *a*.

The apparatus here described is most generally used as a *chronograph* for the measurement of short intervals of time. It is evident that if the beginning and end of the interval to be measured are recorded on the trace by spark marks, *and if the period of vibration of the fork be known,*

then, with a fork of high frequency, very short intervals of time can be accurately measured.

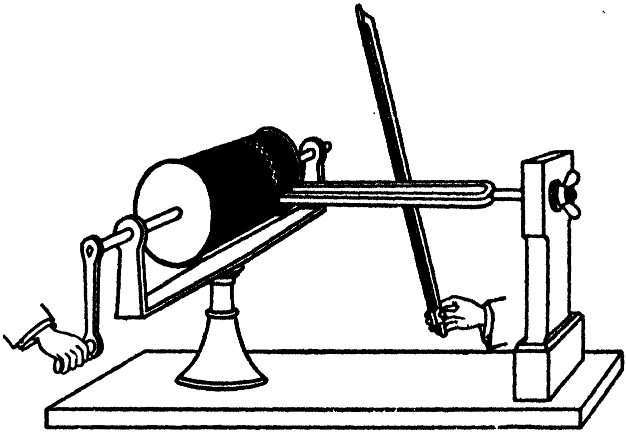


Fig. 63.

Stroboscopic methods have been much used in the more recent determinations of frequency. They are particularly interesting in character and give very accurate results, but the details of their working are generally too complicated for consideration here. The general principles involved in the method may however be illustrated by the experiment described below.

Fig. 64.

Imagine a pendulum beating, say, half-seconds, to be observed through a small hole in a plate, and that the view through this hole is interrupted periodically by the prong of a tuning-fork vibrating close to the hole across the line of sight. If the fork makes, say, about ten vibrations per second, then the observer evidently gets ten glimpses of the pendulum during one complete vibration, and the pendulum therefore, seen in this intermittent way, appears during each vibration in ten separate and distinct positions. If now the fork makes exactly ten vibrations per second

—that is, exactly ten vibrations for each complete vibration of the pendulum—then the ten positions in which the pendulum is intermittently seen will obviously appear to be stationary, vibration after vibration, but if the fork makes slightly more or slightly less than ten vibrations during one complete vibration of the pendulum, then the ten positions will appear to move slowly backward or forward as vibration succeeds vibration. If again in, say, 50 complete vibrations of the pendulum, that is in 50 seconds, the 10 positions move backward or forward through the distance between two adjacent positions, then they are apparently again in the same position and the fork has therefore gained or lost on the pendulum by one complete vibration. That is, in 50 seconds the fork makes $(50 \times 10) \pm 1$ vibrations according as the intermittently seen positions of the fork move backward or forward relatively to the pendulum. The frequency of the fork is therefore either 10·02 or 9·98 vibrations per second.

90. Determination of frequency by comparison with a standard of adjustable frequency. The pitch of a note might conceivably be determined by adjusting the pitch of a note given by a suitably mounted *Savart wheel* or *Seebeck syren* in unison with the given note, and then calculating the frequency of the wheel or syren source from the observed speed of revolution and the number of teeth in the wheel or holes in the disc. It would, however, be extremely difficult to make the adjustment, and considerable elaboration of either instrument would be necessary in order to give anything like an accurate determination. Of the two instruments the syren is by far the most suitable for a purpose of this kind, and it was greatly improved and adapted for determinations of pitch both by Cagniard de la Tour and by Helmholtz.

The usual form of the syren as now constructed is shown in Fig. 65. The disc of the syren, pierced with a ring of holes, is shown at D. The lower and upper ends of its vertical spindle are fixed in suitably placed conical bearings so that the disc can rotate freely, but very close to the plate C, which forms the cover of the wind chest, A. This cover plate is pierced, under the syren disc, with a ring of holes exactly similar in number and position to those in the disc, so that as the disc revolves the holes in the disc come exactly over those in the cover plate as many times during a revolution as there are holes in the ring. The holes in the

disc and in the plate are cut so as to slant in opposite directions, as shown at E in the sectional figure. As a result of these arrangements it will be clear that when air is forced into the wind chest by means of a suitable bellows the syren disc is set in rotation by the rush of air through the ring of holes when the position is that indicated in the

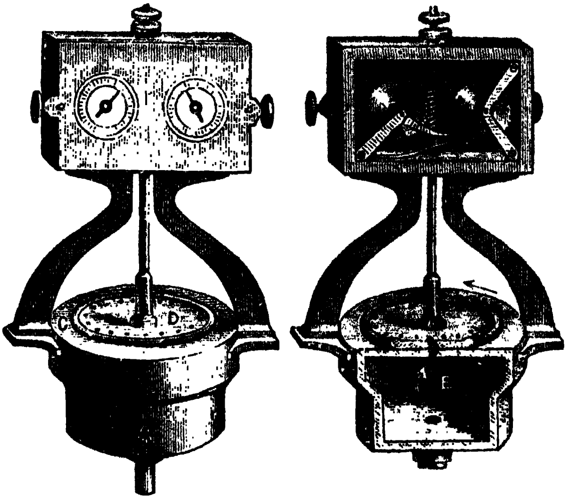


Fig. 65.

sectional drawing. When the disc is in rotation it is clear that, if there are n holes in the ring, then n times in a revolution the holes in the disc will be directly over those in the cover of the wind chest and a ring of air puffs will simultaneously escape from the wind chest. That is, if the disc makes N revolutions per second, then Nn volleys of puffs escape into the air per second and a note of pitch corresponding to this frequency is produced. In order to determine the rate of revolution of the disc a worm on the upper end of its spindle gears with a train of toothed wheels which record the number of revolutions in any interval of time on the dials shown in the figure.

In making a determination of pitch with this instrument the action of the bellows is adjusted until the note emitted by the syren is in unison, or beats clearly and regularly with the note whose pitch has to be determined. When this adjustment is made the train of wheels for recording the speed of revolution is put in gear with the worm on the rotating spindle of the disc for a minute, or for as long as the syren note can be kept constant. The pitch of the note is then given by $Nn \pm x$, where N denotes the observed number of revolutions per second, n the number of holes in the ring, and x the number of beats per second between the syren note and the given note. The positive or negative sign is taken according as the syren note is flatter or sharper than the given note.

This adjustment is a difficult one to make in practice. It is extremely difficult to keep the pitch of the syren note constant for even a short time, and owing to the peculiar quality of the note the adjustment to unison or the observation of beats is generally extremely troublesome. The arrangement by which the disc is driven by the air blast is not a good one; the pressure of the air streaming through the holes tends always to accelerate the motion of the disc and makes it specially difficult to maintain the pitch constant. A much more satisfactory arrangement is to have the disc driven by a suitable motor and to use the air blast only for the production of the musical note. The syren has been very little used in recent experimental work.

A stretched wire or cord may also be used as an adjustable standard of frequency. The method of using it for this purpose is described in detail in Art. **102**.

91. Determination of frequency by comparison with a standard of constant known frequency. The standard source of constant frequency in most general use is the tuning fork. A tuning fork mounted on a properly constructed sounding box, as shown in Fig. 66, is a most satisfactory standard of frequency. The note emitted is pure in quality (Art. **68**), and the pitch is practically constant. Rise of temperature produces a very slight decrease of frequency, but the change is practically negligible.

The pitch of a note can be at once determined by selecting a standard tuning-fork giving a note of approximately the same pitch and counting carefully the number of beats produced in a given time when the two notes are sounded

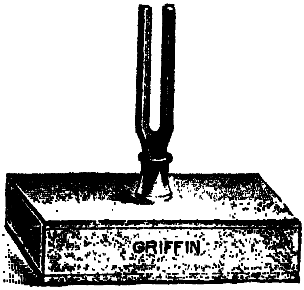


Fig. 66.

together. Then, if N denote the frequency of the fork, and n the number of beats heard per second, the pitch of the given note corresponds to a frequency $(N \pm n)$ according as it is sharper or flatter than the note of the fork.

For the purpose of determining pitch, Scheibler constructed a set of 64 forks covering the range of pitch between the middle C of the piano, of frequency 256, and its octave c, of frequency 572. The forks were adjusted so that the difference between the frequencies of any two consecutive forks of the set was exactly four. With this set of forks, which Scheibler called a *tonometer*, the pitch of any note within the octave could be accurately determined. For example, if the given note gives 102 beats per minute when sounded with the fork of frequency 300, 138 beats per minute with the fork of frequency 304, and 342 beats per minute with the fork of frequency 296, it is obviously sharper than the (300) fork and flatter than the (304) fork, and corresponds to a frequency of 301·7 vibrations per second.

It will be obvious that the method of determining the pitch of a note by comparing it with that of a standard fork is limited by the number and range of standard forks available.

When the frequency to be determined differs considerably from that of the nearest standard available, the comparison may conveniently be made by means of the sonometer method explained in Art. **103**.

Lissajou's method, which can only be referred to here, is another example of this general method. The fork used as a standard of comparison carries the objective of a small compound microscope. The source whose frequency is to be determined is made to vibrate so that a point on it in the field of view of the microscope vibrates at right angles to the plane of vibration of the prongs of the fork. The vibratory motion of the fork and the vibratory motion of the source are in this way compounded at right angles to each other, and the form of the resultant path, as observed through the microscope, indicates the ratio of their frequencies. It also indicates the character of the vibratory motion of the source in such a way as to furnish data from which curves, such as that shown in Fig. 56 for a violin string, can be drawn.

92. Rayleigh's method of determining the pitch of notes of high pitch. A special method suitable for the determination of pitch for notes of high pitch is due to Lord Rayleigh. It has been explained in Art. **59** that, if a source of longitudinal wave motion is arranged in front of a plane reflecting surface, interference takes place between the direct and reflected waves along the normal to the surface through the source of the disturbance. Hence, if a note is sounded, say, by a small whistle of high pitch, some feet in front of a smooth wall, interference effects may be looked for along the normal to the wall through the source of the note. That is, nodes and antinodes caused by the interference are arranged along this line at regular intervals such that the distance between two adjacent nodes or two adjacent antinodes is half the wave length of the note sounded. Hence, if the positions of the nodes or the antinodes can be fixed, and the average distance from node to node or from antinode to antinode measured, the wave length, and therefore the frequency for the note, can be determined. Lord Rayleigh found that by means of a *sensitive flame* the positions of the *antinodes* can be accurately found. A *sensitive flame* is obtained by increasing the pressure of the gas supply to a

suitable pin-hole burner until the long, tapering, steady flame is on the point of becoming unstable or beginning to "roar." A properly adjusted flame of this kind is found to roar at, and for a short distance on each side of, an antinode, but to be quite steady at all other points on the line of interference.

Example. In an experiment of this kind the average distance between the antinodes was found to be 8·5 cms. This means that the wave length in air of the note sounded was 17 cms. The velocity of sound in the air involved in the experiment was calculated to be 34170 cms. per second. The frequency of the source of sound in the experiment is therefore given by 34170/17, or 2010 vibrations per second.

93. The diatonic scale. The sequence of notes which constitute the well-known diatonic scale in music is a series of eight notes extending in pitch from the lowest note of the series to its octave by a sequence of small intervals characteristic of the scale. The *relative* frequencies for the notes of the scale, which may be denoted in the usual way by the letters C, D, E, F, G, A, B, c, are represented by the figures given below:—

C	D	E	F	G	A	B	c
24	27	30	32	36	40	45	48

These figures are merely the *smallest integers* which represent the *relative* frequencies for the notes; they are obviously too small to represent the absolute frequencies of audible notes.

The interval between any two notes is usually measured by the ratio of the corresponding frequencies, so that the sequence of intervals which characterises the diatonic scale is easily found to be as given below:—

C		D		E		F		G		A		B		c
24		27		30		32		36		40		45		48
	$\frac{9}{8}$		$\frac{10}{9}$		$\frac{16}{15}$		$\frac{9}{8}$		$\frac{10}{9}$		$\frac{9}{8}$		$\frac{16}{15}$	

These intervals are evidently not all equal. The interval from C to D, F to G, and A to B, and measured by the ratio 9/8, is known as a *major tone*. The interval from D to E and G to A, measured by the ratio 10/9, is called a *minor tone*. The smaller interval found between E and F and between

B and C, and measured by the ratio 16/15, is called a *semitone*, sometimes a *major semitone*. The sequence of intervals which occur in going up the diatonic scale are therefore: major tone, minor tone, semitone, major tone, minor tone, major tone, semitone; or, omitting the difference between the tones, they are tone, tone, semitone; tone, tone, tone, semitone.

Starting from any note on the scale as the first, the interval to the second, third, fourth, fifth, sixth, or seventh from it is usually called a *second, third, fourth, fifth, sixth,* or *seventh,* as the case may be. The interval from any note to the eighth from it is evidently the *octave.* Thus we have a *second,* measured by the ratio 9/8, from C to D, a *third,* measured by the ratio 5/4, from C to E, a *fourth,* measured by the ratio 4/3, from C to F, a *fifth,* measured by the ratio 3/2, from C to G, a *sixth,* measured by the ratio 5/3, from C to A, a *seventh,* measured by the ratio 15/8, from C to B, and an *octave,* measured by the ratio 2/1, from C to *c*. It will be seen that intervals of the same name are not of exactly the same value in different parts of the scale. Thus, as already explained, a *second* may be a major tone, a minor tone, or a semitone. Similarly, the *third* from C to E, measured by the ratio 5/4, is known as a *major third* and is greater than the *third* from E to G or from A to C, which is measured by the smaller ratio 6/5 and is called a *minor third.* The notation which applies to the various kinds of intervals belongs, however, to music rather than to the science of sound, and need not here be further considered.

The notes C, E, G, *c*, which form what is called the *common chord* of the scale, have frequencies which may evidently be represented by the numbers 4, 5, 6, 8, as below:—

C	E	G	*c*
4	5	6	8

The relative frequencies for these notes should be remembered and also the value of the intervals C to E, a major third, measured by the ratio 5/4; C to G, a fifth, measured by the ratio 3/2; and C to *c*, the octave, measured by the ratio 2/1.

Since the interval between any two notes is measured by the ratio of the frequencies for these notes it follows that any two intervals are added together by taking the *product* of their measures. For example, the interval C to E is measured by the ratio 5/4 and the interval E to G by the ratio 6/5, and the sum of these two intervals, the interval from C to G, should, therefore, be measured by the product $5/4 \times 6/5$ or 3/2, which is obviously in agreement with the measure given by the ratio of the frequencies for C and G. In the same way the difference between any two intervals is evidently got by dividing the measure of one interval into the other. For example, the difference between a major tone and a minor tone is measured by $\frac{9}{8}/\frac{10}{9}$ or 81/80. This small interval is sometimes called a *comma*.

94. Temperament. If we form a diatonic scale on C (256), the middle C of the piano, as *key note* or *tonic*, the frequencies for the notes of the scale will be given, to the nearest integer, by the numbers set out below :—

C	D	E	F	G	A	B	*c*
256	288	320	341	384	427	480	512

The frequencies for notes in octaves above *c* will obviously be multiples of these numbers by 2, 4, 8, 16, etc., and in octaves below the corresponding sub-multiples of the same numbers. If we now again form a diatonic scale on some other note, say E (320), as the *key note* or *tonic* of the new scale we get the frequencies of the scale from E (320) to *e* (640) represented by

E	F	G	A	B	*c*	*d*	*e*
320	360	400	427	480	533	600	640

If this scale is examined it will be seen that within the range of the octave C (256) to *c* (512) it introduces four notes of frequencies 267, 300, 360, 400, which are not found in the scale based on C (256) as key note. Thus, in order that music involving the diatonic scale may be written and read in the key of C and the key of E, notes corresponding to the frequencies 256, 267, 288, 300, 320, 341, 360, 384, 400, 427, 480, 512 are required in the octave C (256) to *c* (512) and corresponding notes in the

octaves above and below this. It will be found on trial that, if we further form the diatonic scale on *each* of the notes in the scale based on C (256) as a key note, a number of new notes will be introduced into the octave from C (256) to *c* (512) by *each* new scale, so that in order to write or read music based on the diatonic scale in any of these keys a *very* large number of notes will be required in each octave. In performing the music the production of these notes might be possible to the human voice or, with sufficient manual dexterity, on the violin, but for any musical instrument dependent upon mechanical manipulation it is quite impossible to provide more than a limited number of notes in each octave. On a piano, for example, the manipulation of a keyboard with a large number of keys in each octave would be a practical impossibility.

For these reasons the diatonic scale has, at least, for the purposes of instrumental music, to be modified or *tempered* so as to admit of modulation in different keys without the introduction of an unmanageable number of notes in each octave.

A number of tempered scales, or *temperaments*, have been proposed, but the simplest is the *equal temperament*, now generally adopted for keyboard instruments such as the piano. On the equal tempered scale the octave is divided into twelve *equal* intervals called semitones. Since these twelve equal intervals added together make up the octave, it follows that if x denote the ratio that measures the interval, then $x^{12} = 2$ or $x = \sqrt[12]{2}$ or, as it is more conveniently written, $x = 2^{\frac{1}{12}} = 1{\cdot}059463$. From this it can readily be calculated that the equal tempered scale from C (256) to C (512) is made up as follows:—

C	. C♯	. D	. D♯	. E	. F	. F♯	. G	. G♯	. A	. A♯	B	. c
	D♭		E♭			G♭		A♭		B♭		
256												572

It will be seen that the scale

C	D	E	F	G	A	B	c
256							572

is made up of the sequence tone, tone, semitone, tone, tone,

tone, semitone, as in the diatonic scale, but the tones are all equal and exactly twice the semitone. The actual frequencies for the notes of the scale differ very little from those of the diatonic scale. On the piano these fundamental notes of the scale C, D, E, F, G, A, B, *c* are given by the white keys, while the *sharps* and *flats*, which are the extra notes necessary for modulation in different keys, are given by the black keys.

It is obvious from the construction of the equal tempered scale that it may be based on any of the notes of a given octave without introducing any new notes.

95. Standards of pitch. The standard of absolute pitch assigned to the middle C of the piano varies slightly, but in England *concert pitch* requires a frequency of 273 for this note. The old *Philharmonic pitch* gave 270 as the frequency for C and the new *Standard pitch* adopted in 1899 requires C and A standard tuning-forks to have frequencies of 261 and 439 respectively at 68° F. For scientific purposes the pitch of C is generally taken to correspond to a frequency of 256. The equal tempered scale from C (261) to C (522) in accordance with this standard pitch is therefore:—

C	C♯	D	D♯	E	F	F♯
261,	276·6,	293,	310·4,	328·9,	348·4,	369·2,

G	G♯	A	A♯	B	*c*
391·1,	414·4,	439,	465·1,	492·7,	522.

EXERCISES XIV.

1. Define pitch, frequency, amplitude, wave-length.

2. Describe one good method of finding the frequency of vibration of a body.

3. Describe the siren. How is it employed to find the frequency of vibration of an organ pipe?

4. A wheel with 33 teeth touches a card as it spins, and thereby emits a note two octaves above middle C, which has 256 vibrations per second. How many revolutions is the wheel making per minute?

5. A cog-wheel containing 64 cogs revolves 240 times per minute. What is the frequency of the musical note produced when a card is held against the revolving teeth? Find also the wave-length corresponding to the note if the velocity of sound is 1126 ft. per second.

6. The disc of a siren contains 40 holes. How many revolutions must it make a second to give a note of frequency 512?

7. How are *beats* used to effect exact unison?

8. How would you measure the frequency of vibration of a body which gives a note beyond the upper limit of audibility?

9. What is the interval between a note and its octave? Check your answer by adding the intervals between the 8 notes of the octave.

10. What is the interval of a major third, a fourth, and a semitone?

11. What is the *common chord* of the scale?

12. If a certain C be produced by 256 vibrations per sec., find the frequency of G, upper G, and lower G.

13. If G be produced by 320·0 vibrations per sec., find the frequency of C in the same octave.

14. What do you understand by temperament? Describe the scale of equal temperament.

EXAMINATION QUESTIONS.

1. A musical note is sounding in a large open space. Describe the nature of the motion of a particle of air a few feet distant from the instrument emitting the note. How would the motion be affected (i) if the pitch were raised to the octave, (ii) if the note became louder, (iii) if it were replaced by another of the same pitch and loudness but of different quality?

2. Describe in detail how you would demonstrate the fact that a shrill note is of great frequency.

Can air-waves of any frequency whatever produce the sensation of sound in the ear?

How has this question been answered experimentally?

3. Describe any laboratory experiment by which you could determine the velocity of sound in air.

How does the velocity of sound in a gas vary (*a*) with the temperature, (*b*) with the pressure of the gas?

4. Describe the changing conditions of a mass of air traversed by a succession of sound waves that proceed from a regularly vibrating source. How does the velocity of such waves depend upon the temperature, pressure, and hygrometric condition of the air?

Mention some observation that indicates that the velocity of a sound wave is independent of its length, and of its amplitude of vibration.

5. Two bells are struck regularly and simultaneously 40 times per minute. At first they are equidistant from an observer, but the distance of one of them is gradually increased. Assuming the velocity of sound to be 1120 feet per second, calculate what the difference between the distances of the bells from the observer will be before he again hears them sound simultaneously.

Why would it be difficult to verify the calculation by direct experiment?

6. How does the velocity of sound in a gas depend upon (i) temperature, (ii) pressure, (iii) the nature of the gas?

How has the velocity of sound in water been determined?

7. Describe a method of determining the velocity of sound in the open air. Point out the sources of error, and explain how some of them may be allowed for.

8. A tuning-fork, vibrating in oxygen gas, produces waves, each four feet in length and travelling with a velocity of 1100 feet per second. Determine its frequency of vibration, explaining the process. How would the velocity, wave-length, and frequency be affected if the same fork were sounding in hydrogen at the same pressure and temperature, the density of oxygen being 16 times that of hydrogen at the same pressure and temperature?

9. How would you determine the number of vibrations per second executed by the prong of a tuning-fork?

10. How would you show that the prongs of a sounding tuning-fork are in vibration, and that the intensity of the note emitted depends upon the amplitude of the vibration?

Describe carefully some method of determining the number of vibrations that take place per second.

11. Describe a siren and explain how it may be used to find the frequency of a tuning fork.

If there are 32 holes in the disc which makes 1050 revolutions per minute, what is the frequency of the note emitted by the siren?

CHAPTER XV.

TRANSVERSE VIBRATION OF STRINGS.

96. Specification of a string. A "string" in acoustics is usually understood to mean a cord or wire or filament of any material. Unless otherwise specified, it is supposed to be uniform in material and section throughout its length, and to be, therefore, of constant mass per unit length. It is further assumed to be perfectly flexible, or rather, its stiffness is supposed to be negligible, and during vibration the changes in length which it undergoes are also assumed to be negligibly small. These conditions are found to be fairly well satisfied by any ordinary cord or wire of not too great thickness, under a sufficiently high pull per unit area of cross-section. The limit of thickness depends on the material, and cannot here be definitely specified; but, as an example, it may be noted that a steel wire of not more than, say, No. 20 gauge fulfils the specified conditions quite satisfactorily. The thin catgut strings of a violin are almost perfectly flexible.

97. The transverse vibration of a string stretched between two fixed points. The mode of vibration of a string as a whole, when stretched between two fixed points, has already been described in some detail in Art. 23. It may, however, be of advantage to recapitulate here the main points of the description.

When set in vibration as a whole, the string vibrates between the extreme curved positions indicated in Fig. 15, so that all points vibrate with the same period and in the same phase, but with different amplitudes. The amplitude increases from zero at each end to a maximum at the

middle of the string, and, as the curve into which the string is displaced approximates to a *sine curve*, the amplitude at any point at a distance x from one end may be expressed, as explained in Art. **23**, by $r \sin 2\pi x/l$, where r is the amplitude at the middle point and l the length of the string.

Vibrating in this way the string is said to vibrate in one *ventral segment* or *loop*, with *nodes* at each end and an *antinode* at the middle. The nodes are evidently points of minimum displacement and maximum flexure of the string (strain), while the antinode is the point of maximum displacement and minimum flexure or strain. This is the simplest mode of vibration of the string as a whole.

The period of vibration of the string can be determined by the method of Art. **4**, by making certain assumptions as to the form of the curve into which it suffers displacement, and then finding the force which acts upon any very short portion of the string as the result of its displacement from its normal position of rest.

Thus, in Fig. 67, if $a\,b$ represent a very small portion of the string, then the force tending to restore it to its original position is the resultant, R, of the tension T acting, as shown at the points a and b, on this portion of the string. If the form of the deflected string is known, the value of this resultant can be obtained and the period of vibration of the portion $a\,b$, and therefore of the whole string, can evidently be determined.

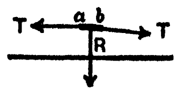


Fig. 67.

This method cannot, however, be further dealt with here.

The period of vibration of the string can, however, be obtained more simply in the following way. Let A B

Fig. 68.

(Fig. 68) represent a string stretched between the points A and B, along which the small lateral deformation $a\,b$ is travelling in the direction indicated by the arrow. This deformation will travel as here represented up to the

end B. At this point it will, as explained in Art. 40, be reflected with reversal of displacements, and travel back towards the end A, as shown in Fig. 69. Here it will again be reflected with reversal of displacements, and then

Fig. 69.

travel towards B as when first considered. It will be seen on consideration that the motion of the string produced in this way is of a periodic character, and that the period of the motion is the time taken by the deformation ab to travel in either direction from any point on the string back through the same point in the same direction. That is, the period of the motion is the time taken by the deformation in travelling twice the length of the string. Hence, if v denote the velocity with which the deformation travels along the string, and l the length of the string, then $2l/v$ is the period of the string's motion. It can be shown, by a method exactly similar to that adopted in Chapter VII., for the proof of the relation $V = \sqrt{E/D}$, that $v = \sqrt{t/m}$, where t denotes the tension and m the mass per unit length of the string, and substituting this value of v in the result obtained above, we get $2l/\sqrt{t/m}$ as the period of the string's motion, and $\frac{1}{2l}\sqrt{t/m}$ as the frequency of its motion.

If, now, we suppose the string AB deformed into one of the curved positions indicated by the dotted curves in Fig. 15, and imagine this deformation, that is, the sequence of displacements which determine it, to travel backwards and forwards along the string by repeated reflexion at the fixed ends, it will be seen that the periodic motion considered above becomes identical with the vibration of the string as a whole as already described.

It will also be clear that the period of vibration will be the time taken by the deformation, that is, by any particular displacement in it, to travel over twice the length of

the string, and that the frequency of the string's vibration is given by $n = \frac{1}{2l}\sqrt{t/m}$.

If the more general assumption is made, that the deformation of the string into one of its extreme curved positions is resolved, as explained in Art. 40, into two component deformations travelling in opposite directions along the string, then the stationary vibration of the string may be supposed to be the result of the interference between these two component deformations which travel, by repeated reflections at the ends, backward and forward along the string in opposite directions.

Again, if a portion of the string, represented by A X (Fig. 70), of length x be taken and fixed at the point X,

Fig. 70.

it can obviously be set in vibration as a whole, as a string of length x with frequency $\frac{1}{2x}\sqrt{t/m}$. Further, if x is an aliquot part of l, the vibration of the portion A X will set the remainder of the string, X B, in sympathetic vibration, and the whole string will vibrate in segments of length x.

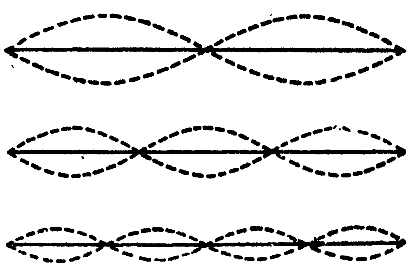


Fig. 71.

For example, if x is one-half, one-third, or one-fourth of l, then the string vibrates in two, three, or four segments, as indicated by the dotted curves in Fig. 71. That is, if $x/l = p$, where p is an integer, then the whole

string A B will vibrate in p segments, and the frequency of this mode of vibration will be $\frac{1}{2x}\sqrt{t/m}$ or $\frac{p}{2l}\sqrt{t/m}$. This result shows that for any given string the frequency of vibration is directly proportional to the number of segments in which it vibrates.

It should be noticed that when a string vibrates in segments, each segment vibrates in the mode already described for a string as a whole, but that the phase of vibration in adjacent segments differs by half a period. That is, adjacent segments of a string vibrating in segments are always in opposite phases.

If the vibration of a string in segments is assumed to be initiated by first setting an aliquot part of the string in vibration, the vibration of the string in segments may be explained as follows. The vibration of the part, A X, first excited is a source of transverse waves, of wave-length $2x$, which travel along the rest of the string from X to the end B, where they are reflected back to meet the direct waves. The interference between these trains of direct and reflected waves, of wave-length $2x$, sets this portion of the string in stationary vibration in segments of length x in proper phase accord with the portion A X, and the whole string is thus set in vibration in segments of length x.

When the string vibrates as a whole (in one "segment") it is vibrating in its simplest mode, and the note which it emits is called the *fundamental note* of the string. When it vibrates in two segments, the frequency is twice that for the fundamental note, and the note emitted is the octave of the fundamental note. This note is usually called the *first harmonic*. Similarly, when the string vibrates in three segments, the frequency for the note produced is three times that for the fundamental note, and the pitch of this, the *second harmonic*, is a fifth above the first harmonic. It may, in fact, be stated as a general result that when a string vibrates in n segments it gives a note for which the frequency is n times that for the fundamental note, and this note is the $(n - 1)$th harmonic.

The modes of vibration of a string in segments by which it gives the harmonics of its fundamental note are sometimes called the *harmonic modes* of vibration of the string.

98. Compound modes of vibration of a string. In general, when a string is bowed or excited in any way, it does not vibrate in any *one* of the modes described above, but in a mode compounded of some or all of the modes of which it is capable. In fact, it is extremely difficult to so excite a string as to set it in one definite mode of vibration. One particular mode may be *predominant*, but the vibration is in general the resultant of all the modes of which the string is capable under the conditions of its vibration. This compound mode of vibration of the string will be more easily understood if it is realised that in the compound mode the motion of any point on the string is merely the resultant of all the motions proper to the component modes present.

Hence, when a string is apparently sounding its fundamental note or a particular harmonic, this note is generally only the predominant component of a compound note made up of all the harmonics (including the fundamental note) possible under the conditions of vibration. It is the presence of these harmonics which determines the *quality* of the note

99. Experimental study of the harmonic modes of vibration of a stretched string. Let a string be stretched between two wrest pins on a board, and let it pass over and rest upon two knife-edged rests or *bridges* fixed one at each end just in front of the pins, so as to give between them a definite length of string. For the purposes of vibration the string is "fixed" at the points where it rests on the bridges, and the length of the string in vibration is therefore exactly given by the distance between the bridges.

If the string is bowed at the centre it can be seen to vibrate as one loop of segment, as explained above.

If it is bowed at a point at a distance of a quarter of the length of the wire from one end the predominant vibration

will be vibration in two segments or loops. If while bowing it the string is also touched at the middle point, as shown in Fig. 72, the vibration in two segments can be plainly seen. Even when the amplitude of vibration is very small the existence of an antinode at the middle of the unbowed segment can be demonstrated by placing a small V-shaped paper rider at this point on the string before bowing. When the string is bowed the vibration set up "throws" the rider, as shown in Fig. 72. It will be found that even if the string is held firmly at the middle point both

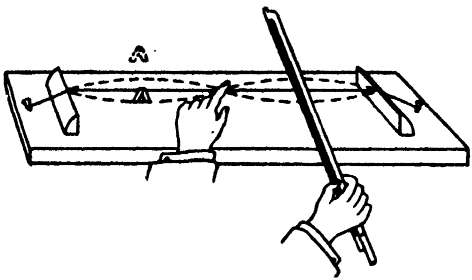


Fig. 72.

segments are set in vibration by bowing one of them at its middle point. This shows that there is transmission of energy through the node from one segment to the other. It is also an illustration of the principle of resonance, for, since the period of vibration for each segment is the same, the impulses transmitted through the node from the segment are timed exactly as they should be to set the other segment in vibration.

If the string is bowed at a point distant one-sixth of the length of the string from one end, the string should vibrate in three segments, as shown in Fig 71. In order to set up this mode of vibration strongly it is necessary to touch the string at a point distant one-third of the length of the wire from one end and to bow carefully at the middle of the short segment. The vibration in three segments thus set up is conveniently exhibited by placing riders at the

points where nodes and antinodes should be formed. Then on bowing the string it will be found that the riders are "thrown" at the antinodes, but remain in position at the nodes, as represented in Fig. 73. Generally, if the string is bowed at a point $l/2n$ from one end and touched, so as to form the first node, at a point l/n from that end, then the string tends to vibrate in n segments, and, if n is not too great for the wire, the nodes and antinodes of n segments can be shown by means of riders, as explained above.

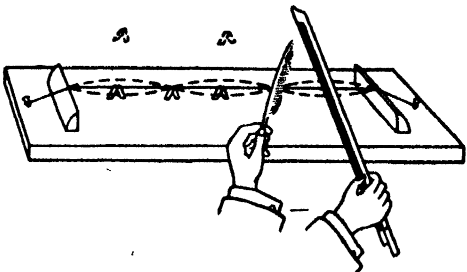


Fig. 73.

100. Experimental study of the compound modes of vibration of a string. It has been explained in Art. 98 that a string, when bowed, tends to vibrate, not in one mode only, but in a complex or compound mode which, as explained above, is really the resultant of all the modes practically possible to it under the conditions of vibration. For this reason it is necessary, in order to make the required mode of vibration at least the predominant one, not only to bow the wire at the right point, but also to determine the adjacent node by touching or *damping* the string with a light pointer, such as a quill or pencil or the edge of a ruler or paper knife, at the point where the node should form.

The fact that the usual mode of vibration of a string is the resultant of all the simple modes possible under the conditions of its vibration can be readily detected by the musically educated ear. Thus, if the compound note

given out by a vibrating string be carefully listened to, a competent observer can detect not only the fundamental tone, but also the first harmonic an octave above the fundamental, the second harmonic a fifth above the first, the third harmonic an octave above the second, and possibly some higher harmonics. The presence of these harmonics in the compound tone indicates the presence of the corresponding mode of vibration as a component of the compound vibration of the string. These harmonics are more easily detected by the ordinary observer by means of *resonators* consisting of tubes or hollow globes of such dimensions that they form a set of which the members individually respond, by resonance, to the harmonics for a particular fundamental note. If, therefore, a string is tuned until its fundamental note excites the first resonator to loud resonance it will be found that the other resonators of the set are also excited to resonance, and that any particular harmonic present can be distinctly heard by putting the resonator which responds to it to the ear. Fig. 74 shows one of a set of resonators of the type used by Helmholtz for the analysis of complex sounds by this method.

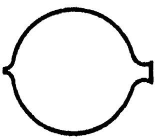


Fig. 74.

The existence of any particular harmonic mode in the compound vibration of a string can also be exhibited by a simple experiment on the string itself. If a string of length l while in compound vibration is damped lightly at a point distant l/n from one end, the node at that point is emphasised and the $(n-1)$th harmonic at once becomes predominant and may be heard distinctly. For example, if the string is damped at a point one-third of the length of the string from one end, the second harmonic, or twelfth above the fundamental note, at once becomes audibly apparent as a component of the note given by the string.

If a string is damped effectively at the exact point, which should be an antinode for some of its modes of vibration, and then bowed, the complex vibration of the string will not include these particular modes of vibration. That is, the compound note emitted by the string will include only

some of its harmonics. For example, if the middle point of a string is effectively damped, its compound mode of vibration when bowed cannot include any of the modes requiring the string to vibrate in an *odd* number of segments as its components. That is, the note given by the string is compounded only of the *even* harmonics of the string. Since the quality of the note is said to depend on the number, order, and relative intensity of the harmonics present, the quality of the note given by a string damped at its middle point should differ from that given by the free string. It will be readily recognised on comparing the notes that this is the case.

In the same way, if a string is bowed exactly at a point which should be a node in certain modes of vibration of the string, then the compound mode of vibration set up by the bowing will not include these particular modes. For example, if a string is bowed carefully at the exact middle point (and at the same time slightly damped at a point one-third or one-fifth of the length of the string from one end), all the modes of vibration which divide the string into an even number of segments, and which therefore require a node at the middle point, will not be included among the components of the compound mode of vibration set up in the string. That is, the note given out by the string is compounded only of the *odd* harmonics of the string, and the quality of the note will therefore differ from that given out by the free string or by the string when damped at its middle point.

101. Experimental study of the conditions which determine the period of vibration of a stretched string. For the purpose of these experiments an elaboration of the apparatus shown in Figs. 72, 73, and usually known as a *monochord* or *sonometer*, is required. As generally constructed the sonometer consists of a sounding box A A, mounted as shown in Fig. 75, and carrying the stretched strings to be experimented with on the covering board of this box. The instrument is usually arranged to carry two strings. As shown in the figure, one string G G can be stretched in the usual way between two wrest pins over fixed terminal

bridges. The other string F F is attached to a wrest pin at one end, but passes over a pulley wheel at the other end and carries at its extremity a scale pan or weight carrier for carrying weights, as shown at E. By means of these

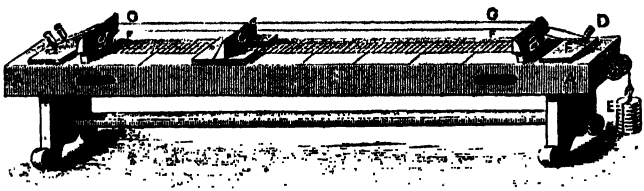


Fig. 75.

weights tension is applied to the string and can be adjusted and measured as required. This string also passes over the fixed terminal bridges B and B′, but the bridge near the pulley wheel must be so adjusted as not to cause a change in the tension of the string at the point where it rests on the bridge. The string F F, to which tension is applied by means of weights, is the one to be made the subject of experiment. The string G G, stretched between the wrest pins, is usually required for purposes of reference as to pitch, and can be adjusted for pitch by altering its tension by means of a key applied to the wrest pin at D. It is convenient to use thin pianoforte steel wire for this string. The fixed length of the strings in vibration is measured by the distance between the terminal bridges B B′, and this distance, usually a metre, or half a metre, is divided by a scale marked on, or fixed to, the cover board of the sounding box, as shown in the figure. In order to experiment on different lengths of either string a *movable* bridge C is also provided. This bridge can be moved backward and forward along the length of the string, and by means of the scale any required length of either string can be set in vibration. It is convenient if this bridge is so constructed that either string may be damped at any point without affecting the other.

A simple and convenient sonometer is obtained by fixing a sounding board vertically on a wall and stretching the strings vertically, as shown in Fig. 76, from pins or hooks at the top of the board over two movable bridges. The tension is readily applied by hanging weights on the strings.

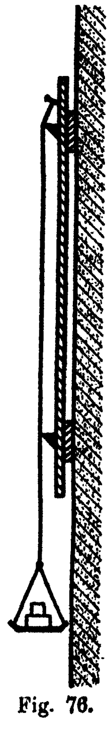


Fig. 76.

The only factors on which the period of vibration of a string or wire, stretched between two fixed points, may possibly depend are the length, the tension, the form and dimensions of the cross section, and the material of the string. There are no measurable quantities relating to the string which are not included under these heads. The string, however, is supposed to be perfectly flexible, or at least of negligible stiffness, and the small changes of length which it undergoes during vibration are also negligible. The elastic properties of the material, and the *form* of the cross section of the string need not therefore be considered, for, with these assumptions, they can have no influence on the period of vibration. In fact, under these conditions, the problem reduces to the comparatively simple one of the dynamics of the motion of the vibrating string, and the only factors which have a bearing on the frequency of vibration are the length, the tension, and the mass of the string. If the mass of the string is uniformly distributed along its length, that is, if the mass per unit length is constant, then the three quantities on which the frequency of vibration of the string depends are the length in vibration, the tension, and the mass per unit length.

It should be noted here that the problem of determining the laws of vibration of a stretched string is not essentially an experimental one. It is really a problem in dynamics and can be solved *theoretically* by application of the principles of dynamics without recourse to experiment.

The theoretical solution, as explained above, gives the result that the frequency of vibration, n, is given by

$$n = \frac{1}{2l}\sqrt{t/m}$$

and indicates at once that the only quantities involved are: the length in vibration, l; the tension, t; and the mass per unit length, m. The function of experiment in this case is really to verify this relation, and to determine how far the results of experiment justify the assumptions made in obtaining the result.

The problem may, however, be studied experimentally with advantage. In order to determine by experiment how the frequency of vibration of a stretched string depends on *each* of the factors which *jointly* determine the frequency, the usual experimental method which applies to such an investigation has to be adopted. A number of experiments are made in sets, in each of which one factor, taken separately, is varied while the others remain constant. It may be assumed, as explained above, that the factors on which the frequency of vibration of a string depends are three—the length in vibration, the tension, and the mass per unit length. In order to determine how the frequency depends upon (1) the length, (2) the tension, and (3) the mass per unit length, *three* sets of experiments are required. These are (1) experiments in which the length is varied while the tension and mass per unit length are kept constant, (2) experiments in which the tension is varied while the length and mass per unit length remain constant, (3) experiments in which the mass per unit length is varied while the length and tension remain constant.

These experiments are most conveniently made by means of the sonometer. A few typical experiments of the first set, for the determination of the relation between the frequency and the length of the string, are given below.

Exp. 27. Adjust the tension of the string stretched between the two wrest pins until it gives a note of convenient pitch, say about G (384) or C (256). Sound the note when adjusted and note the pitch as a key note. Now, by means of the movable bridge, mark off one half the length of the string and set it in vibration; the note

heard will be at once recognised as the octave of the key note. That is, by halving the length of the string without altering the tension or mass per unit length the frequency is doubled. Similarly, if a third of the length of the string is set in vibration, the note heard will be recognised as the fifth above the octave of the key note: that is, when the length of the string in vibration is divided by three, the frequency of vibration is multiplied by three. In this way it can be established generally that if the nth part of the string is set in vibration the frequency is n times that of the key note. This indicates that *the frequency varies inversely as the length* of the string in vibration when the tension and mass per unit length are kept constant. This result may be further confirmed by noting that if we take four-fifths of the length of the string we get the *third* above the key note, a note for which the frequency is five-fourths that of the key note. Similarly, if we take two-thirds of the length, we get the fifth above the key note with a frequency one and a half times that of the key note. In this way, if we take in succession lengths $\frac{8}{9}$, $\frac{4}{5}$, $\frac{3}{4}$, $\frac{2}{3}$, $\frac{3}{5}$, $\frac{8}{15}$, and $\frac{1}{2}$ that of the full length of the wire, we get the succession of notes which constitute the diatonic scale on the key note as tonic.

Exp. 28. In the experiments described above the tension and mass per unit length are kept constant by experimenting throughout with different lengths of the *same* string. If, however, we take two strings of different material and differing in diameter and, if desired, in form of cross section, but having *the same mass per unit length*, and stretch them, in succession over the pulley of the sonometer with *the same tension*, it will be found that for equal lengths they give out the same note when sounded. It will also be found that the note given by the nth part of either wire, whether n be integral or fractional, has a frequency n times that of their fundamental key note. In making the comparison the reference string of the sonometer is tuned into unison with the first string stretched over the pulley, and the note afterwards given by the second string is compared with that of the reference wire. This result shows that even when the mass per unit length is kept constant by varying the material and cross section of the wire the law that frequency varies inversely as the length still holds. This shows that mass per unit length is the determining factor, and that material and cross section enter into the question only in so far as they affect the mass per unit length.

The general result of these experiments is that the frequency of vibration of a stretched string varies inversely as the length in vibration when the tension and mass per unit length of the string are kept constant. That is, n *varies as* $1/l$ *when* t *and* m *are constant.*

The experiments of the second set, for the determination of how the frequency depends upon the tension when the length and mass per unit length are kept constant, are of the type indicated below.

Exp. 29. Stretch the string to be experimented with on the sonometer over the pulley wheel, and adjust the tension by applying weights until the string gives a note of convenient pitch. For purposes of reference, tune the note of the other string into unison with this note. Now add weights so as to increase the tension, and note that the frequency increases as indicated by the rise in the pitch of the note given. Continue this increase of tension by adding weights until the pitch of the note given by the string has risen exactly an octave,* and is therefore adjusted to be exactly an octave above the note of the reference wire. It will now be found that the tension is *four* times what it was to start with, that is, an increase of tension in the ratio 1 : 4 gives an increase in frequency in the ratio 1 : 2.

If the tension is now further increased until the fifth above this octave is obtained, it will be found that the tension is *nine* times its first value, that is, an increase of tension in the ratio 1 : 9 gives an increase of frequency in the ratio 1 : 3. By this means it can be established as a general result that an increase of tension in the ratio 1 : n gives an increase of frequency in the ratio 1 : $\sqrt{n}$, or more generally, if the tension varies in the ratio $a : b$ then the frequency varies in the ratio $\sqrt{a} : \sqrt{b}$. That is, the frequency varies directly as the square root of the tension when the length and mass per unit length remain constant.

Thus, if the tension is made to vary in the ratio 9 : 4 the frequency varies in the ratio 3 : 2 and the two corresponding notes are a fifth apart; if the tension is varied in the ratio 16 : 25 the frequencies vary in the ratio 4 : 5 and the two corresponding notes are a third apart.

In the experiments described above the length and the mass per unit length are kept constant by experimenting throughout with the same length of the same string under varying tension. It will be found, however, that the same results will be obtained with string differing in material and cross section so long as the length and *mass per unit length* are kept constant and only the tension varied.

* For the purpose of this experiment, and many other similar ones, it is convenient to replace the usual scale pan or weight carrier by a stout water can, and to add weight by pouring water into the can from a measure graduated in c.cms. or ounces.

The general result of this set of experiments is, therefore, that the frequency of vibration of a stretched string varies inversely as the square root of the tension when the length and mass per unit length of the string are *kept constant*. That is, *n varies as $\sqrt{t}$ when m and l are constant.*

The third set of experiments for the determination of the relation between the frequency and the mass per unit length, when the length and tension of the string remain constant, has now to be considered. It will be realised that, in these experiments, the mass per unit length cannot, like the tension or length of the string, be adjusted conveniently in any required ratio. All that can be done is to take a number of strings of different mass per unit length and after determining the exact value of this quantity for each string, by carefully weighing a measured length of the string, to find by experiment the relation between the frequencies of the strings.

Exp. 30 Take any two strings (either of the same material or of different material) of different mass per uni length and determine as accurately as possible the mass per unit length of each string by weighing a measured length, say a metre, of the string and calculating the required result. Let the masses per unit length of the strings be denoted respectively by m, and m', of which m is the greater. Now stretch the string having the greater mass per unit length on the sonometer over the pulley wheel, and adjust the tension until the string gives a note of convenient pitch a little higher than the note given by the reference string. By means of the movable bridge, the length of the reference string which gives a note of exactly the same pitch as the string under experiment can now be found. Let this length be denoted by l. Remove this string and stretch the other in its place, under exactly the same tension; then, as before, find the length of the reference string which gives a note of exactly the same pitch as that of the stretched string. Let this length be denoted by l'. Then, as the notes given by the strings are in unison with the notes given by lengths l and l' of the reference string, the frequencies of the string must, in accordance with the law determined by the first set of experiments, be in the ratio $l' : l$. That is, if n and n' denote respectively the frequencies of the strings of which the masses per unit length are denoted respectively by m and m', then $n/n' = l'/l$. The relation between the ratio m/m' and the ratio l'/l, both of which have been found by experiment, can now be investigated. It will always be found that for any two strings the relation is that expressed by writing $n/n' = \sqrt{m'/m}$. That is, the frequencies of the strings are

inversely proportional to the square roots of their masses per unit length, and this is true not only for strings of the same material, but generally for strings of different materials.

The general result of this third set of experiments is, therefore, to establish the relation that the frequency of vibration of a stretched string varies inversely as the square root of the mass per unit length, when the length and tension of the string are kept constant. That is, *n varies as $1/\sqrt{m}$ when l and t are constant.*

Three general results have now been established by means of the three sets of experiments described above. These results are sometimes called the laws of vibration of a stretched string. As already indicated, they are conveniently expressed by the three relations given below.

(1) *n varies as $1/l$ when t and m are constant.*

(2) *n varies as $\sqrt{t}$ when l and m are constant.*

(3) *n varies as $1/\sqrt{m}$ when l and t are constant.*

If these three results are combined algebraically, we get the general result denoted by writing—

$$n \text{ varies as } \frac{1}{l}\sqrt{\frac{t}{m}}.$$

That is,

$$n = k\,\frac{1}{l}\sqrt{\frac{t}{m}},$$

where k is a constant. If the frequency of a string, for which l, m, and t are known, is determined directly by one of the methods given in Chapter XIV., then the value of k can be determined by substituting the known values of n, l, t, and m in the above relation. It will be found that for any string for which this determination is made the value of k is $\frac{1}{2}$. We can therefore write, as the result of an experimental investigation, that

$$n = \frac{1}{2l}\sqrt{\frac{t}{m}}.$$

This is the result already obtained theoretically in Art. 97.

There are one or two corollaries which may be deduced from the relation between n and m. If a denote the area of cross-section of the string, then we obviously have $m = ad$, where d denotes the *density* of the material of the string. Also, if s denote the tension per unit area of cross-section, or stretching stress, then $as = t$, and by substituting the values thus obtained for m and t in the relation given above, we get

$$n = \frac{1}{2l}\sqrt{\frac{s}{d}},$$

which implies that n varies inversely as l, directly as $\sqrt{s}$, and inversely as $\sqrt{d}$.

Again, if the cross-section of the string is circular and of radius r, then $m = \pi r^2 d$. If we substitute this value of m in the fundamental relation, we get

$$n = \frac{1}{2lr}\sqrt{\frac{t}{\pi d}},$$

which implies that n varies inversely as l, inversely as r, directly as $\sqrt{t}$, and inversely as $\sqrt{d}$.

These results are, however, merely variants of the fundamental relation,

$$n = \frac{1}{2l}\sqrt{\frac{t}{m}},$$

which expresses concisely the three "laws" for the transverse vibration of strings.

102. Determination of pitch by means of a stretched string. If the string used is of such flexibility that the theoretical relation,

$$n = \frac{1}{2l}\sqrt{\frac{t}{m}},$$

is practically true, the sonometer may be used for the absolute determination of pitch. The string may be stretched on the sonometer over the pulley, and its tension adjusted until a convenient, and not too short, length of it, determined by means of the movable bridge, is found to give a note in unison with that of which the pitch has to be found. If l denote the length thus determined, t the tension applied, and m the mass per unit

length of the string, then the pitch to be determined evidently corresponds to a frequency given by

$$n = \frac{1}{2l}\sqrt{\frac{t}{m}},$$

for which l, t, and m are known.

For the purpose of determining absolute pitch, however, much more accurate results are obtained by letting the string hang freely in a vertical position under a tension determined by the weight carried by the string. A portion of the string is then marked off between two massive clamps, and by means of this portion the pitch of the given note is determined by the method described above For example, in an experiment of this kind, a length of 33 cms. of a string under a tension of 4 kilogrammes weight, and for which the mass per unit length was ·0106 gramme per cm., gave a note in unison with the note whose pitch had to be determined. From these data we have $l = 33$ cms., $t = 4000 \times 981$ dynes,* and $m = \cdot 0106$ gramme per cm., and substituting these values in

$$n = \frac{1}{2l}\sqrt{\frac{t}{m}}$$

we get the frequency corresponding to the given pitch as

$$n = \frac{1}{66}\sqrt{\frac{4000 \times 981}{\cdot 0106}} = 291{\cdot}4.$$

That is, the pitch to be determined corresponds to a frequency of 291·4 vibrations per second.

103. Sonometer methods of comparing pitch. A note of unknown pitch may be conveniently compared with that of a standard fork of higher or lower pitch by means of a sonometer wire. A length, l, of the wire is adjusted so that the note it emits is in unison with the fork; the length is then altered, either by shortening or lengthening it, until, for a length l', its note is in unison with the

* The tension must always be expressed in force units—*dynes* or *poundals*.

given note. The frequencies corresponding to the lengths l and l' of the wire are, as indicated by the formula given above, inversely proportional to these lengths, so that the ratio of the frequency of the source of the unknown note to the frequency of the fork is as l is to l'.

For example, if the note of the fork is found to be in tune with that given by a length of 42 cms. of the wire, and the given note in tune with that emitted by a length of 56 cms. of the wire, then the frequency for the given note is to the frequency of the fork in the ratio 3 to 4, and if the frequency of the fork is known to be 256 vibrations per second, then the required frequency is 192 vibrations per second. This is a convenient method for rough determinations, but it is not capable of great accuracy.

Another convenient method for determining the pitch of a given note by means of a sonometer wire is as follows. Find the length of the wire which gives a note in unison with the given note; let this length be denoted by l. Then find the length (longer) which gives a note which beats with the given note, say, three times a second; let this length be denoted by l'. Then, if n denote the frequency for the given note, we evidently have the relation

$$n/(n-3) = l'/l,$$

from which n can be calculated.

For example, the note of which the pitch was to be determined was found to be in unison with the note given by 40 cms. of the wire, and to make 10 beats in 3 seconds with that given by 42 cms. of the wire. From this we get

$$n/(n-3\tfrac{1}{3}) = 21/20 \text{ or } n = 70.$$

It will be clear that this method is most suitable for notes of low pitch; when n is too great the values of l and l' are so nearly equal that very accurate measurement would be necessary to get the true value of their ratio. There is also some difficulty in making an accurate determination of the number of beats per second. These methods are, in fact, convenient for rapid approximate determination of pitch, but are not suitable for accurate work.

104. Melde's experiments. Some particularly interesting experiments illustrating the transverse vibration of strings are due to Melde.

In these experiments one end of a light silk cord or thin wire was attached to the extremity of one prong of a large massive tuning-fork, and the other end passed over a pulley wheel and carried a light scale pan or weight carrier as on the sonometer. The position adjustments were made so that the cord was in this way stretched horizontally between the prong of the fork and the pulley wheel.

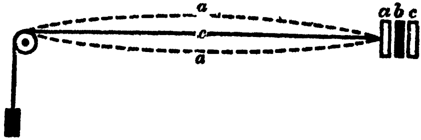


Fig. 77.

In one set of experiments the length of the cord was adjusted in the plane of vibration of the fork, as shown in Fig. 77. In another set the length of the cord was arranged at right angles to the plane of vibration of the prongs (as shown in Fig. 78). In both these figures the fork, whose prongs are shown in cross section, is supposed to be fixed in a horizontal position.

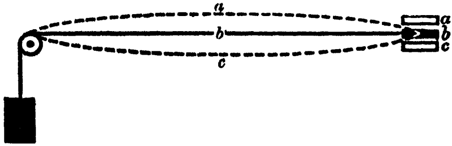


Fig. 78.

It will be clear that if, in either of these positions, the tension is properly adjusted vibration of the string may be set up and maintained by the vibration of the fork.

In the arrangement of Fig. 77 the string, having necessarily the shortest range when in the curved position and the longest range in the straight position, must, when maintained in vibration by the fork, move from the position *a*, through the straight position, *c*,

down to the lower position, a, while the prong of the fork moves from the position a to the position c, and back again to a. That is, the string makes half a complete vibration while the fork makes a complete vibration, and the frequency of the string when maintained in vibration as a whole by the fork is half the frequency of the fork. In order that this mode of vibration may be possible the tension of the string must evidently be adjusted so that the natural frequency of the string under the tension is half that of the fork.

If this tension is increased the natural frequency of the string rises, and as it cannot adjust its length in any way to vibrate in the necessary accord with the *fixed* frequency of the fork, the tension for the adjustment indicated in Fig. 77 is the greatest tension under which the string can be maintained in vibration by the fork.

On the other hand, if the tension is reduced below this limit to one-fourth of its value, the fundamental frequency of the string vibrating *as a whole* will be one half of what it was, so that by vibrating *in two segments* and so doubling its frequency it can again vibrate in the necessary accord with the fork. Similarly, by reducing the tension to one ninth, one sixteenth, one twenty-fifth, and so on, of its initial value, the string can vibrate in perfect accord with the fork by splitting up into three, four, five, or more segments. In this way the tension necessary to make a light string vibrate in six or eight segments may evidently be so small that the damping effect of its attachment to the prong of the tuning-fork may be comparatively small. The end of the string attached to the prong of the fork is not quite fixed, but it has no transverse motion, and its longitudinal motion, determined by the amplitude of vibration of the prong, is small.

In the case of Fig. 78 it will be seen that when the accord necessary for maintenance of vibration is obtained between the natural frequency of the string and the frequency of the fork, the two frequencies must be equal. The string makes a complete vibration through the positions a, b, and c, and back again in the same time as the fork makes a complete vibration through the corresponding positions.

The string will therefore be maintained in vibration as a whole by the fork if the tension is so adjusted that its natural fundamental frequency is the same as that of the fork. For the same string, therefore, the tension for this adjustment must be four times that necessary for the corresponding adjustment for the arrangement of Fig. 77.

In the same way, as explained above, it will be clear that if this tension is reduced to one-fourth, one-ninth, one-sixteenth, etc., of its initial value, the string vibrates in accord with the fork by dividing into two, three, four, or more segments. The point of attachment of the string to the fork in this case is not exactly a node, but a point a little in front of the node at which the amplitude of the transverse vibration of the string is the same as that of

the prong of the fork. The real position of the node is shown at n, and the effective length of the string, $w\,n$, is slightly greater than its actual length.

Melde's experiments may be considered in another and rather instructive manner. They are really simple illustrations of the principle explained in Art. 59 and illustrated by Fig. 47.

The vibration of the prong of the fork acts as a source of transverse waves, which are transmitted along the stretched string to the pulley wheel at w and there reflected back again along the string. Interference takes place between these trains of direct and reflected waves, and, under proper conditions, the string is thereby set in *stationary vibration*.

In the arrangement of Fig. 77 the frequency of the prong *as a source of transverse waves along the string* is n, where $2n$ is its actual frequency of vibration. The velocity of the waves along the string is not constant, but being given by $v = \sqrt{t/m}$ it varies directly as the square root of the tension. Now, if λ denote the wave-length of the transverse waves in any case, we have $v = n\,\lambda$, or $\lambda = v/n$. Hence, if the tension of the string is so adjusted that v has a value such that $\lambda/2$ is equal to the length of the string or to some aliquot part of it, the interference between the direct and reflected travelling waves sets up stationary vibration of the string as a whole or in a definite number of segments of length $\lambda/2$. That is, if $\lambda/2$ or $v/2n = l/p$, where l denotes the length of the string and p is an integer, then the string takes up stationary vibration in p segments.

In the arrangement of Fig. 78 the wave-length of the travelling waves will evidently be $v/2\,n$, and if $v/4\,n = l/p$ as above, then the interference between the direct and travelling waves will set the string in stationary vibration in p segments.

It should be noticed that these experiments are exactly analogous to the much more familiar experiment described in Art. 115. In this experiment the adjustment to obtain stationary vibration is made by adjusting the length of the air column. The adjustment in Melde's experiments can also be made by adjusting the length of the string under a constant tension.

105. Use of strings in musical instruments. The production of musical notes by the vibration of strings finds many important applications in musical instruments. The pianoforte, all stringed instruments of the type of the violin, the harp, the guitar, and other similar instruments,

are examples of instruments on which musical notes are produced by the vibration of strings set in vibration by different methods.

A slight examination of any one of these instruments will show that the laws of vibration of strings are consistently applied in its construction.

In the pianoforte the "strings" are thin steel wires set in vibration by the stroke of soft leather-covered hammers actuated by the keys. The wires for the high notes are short and thin, and under high tension. Those for the low notes are longer and thicker, but as the tension cannot be made very low if the wire is to be kept taut, the mass for unit length has to be further increased for the lowest notes by a covering of fine wire wound spirally round the stretched wire. This gives the necessary increase of the mass of the string without unduly decreasing the flexibility or requiring the tension to be too low for satisfactory action. The hammers too are adjusted to strike the wires at points which render the production of certain undesirable harmonics impossible.

In the violin the strings are of the same length, but the mass per unit length differs with the pitch of the string, and in tuning the adjustment is made by adjusting the tension. Also, in playing the instrument, the note given by any string is varied by adjusting its length, and, when necessary, harmonics are produced by bowing and damping at the right points as described above.

EXERCISES XV.

1. Describe experiments which illustrate the laws of transverse vibration of strings.

2. Two equally stretched strings of the same thickness, one of steel, the other of catgut, give the same note when struck. Which of them is the longer? Give reasons for your answer.

3. What variety of notes can you get out of a stretched string without altering its tension or length? What will be the effect of halving its length by a fixed bridge?

4. Explain the nature of the vibration of a stretched wire. What effect is produced by touching it at one-third its length from end to end?

5. State in what way the rate of transverse vibration of a stretched string depends upon the tension. How would you determine the rate of vibration of the string?

6. Explain the use of the monochord. How may it be employed to exhibit the relations which exist between the best known musical intervals?

7. Explain how the sonometer may be used for the determination of frequency.

8. Two similar strings on a sonometer are tuned to unison. One is 36 inches long and stretched by 100 lbs. Find the weight on the other one, which is 45 inches long.

9. Two similar strings are in unison. One is 36 inches long and stretched by 100 lbs. The other is stretched by 220 lbs. Find its length.

10. One wire is 100 inches long and bears a weight of 200 lbs. Another similar wire yielding the second higher octave of the first bears a weight of 130 lbs. Find the length of this wire.

11. A steel wire 30 inches long and $\frac{1}{50}$ in. diameter yields a note with a frequency of 200 vibrations per second. A second steel wire bears the same weight as the first, but is 20 inches long and $\frac{1}{40}$ in. diameter. Find the frequency of its fundamental note.

12. A steel wire one yard long, and stretched by a weight of 5 lbs., vibrates 200 times per second when plucked. If I wish to make two yards of wire vibrate twice as fast, with what weight must I stretch it?

EXAMINATION QUESTIONS.

1. In what way is the frequency of transverse vibration of a stretched wire affected (*a*) by halving the length of the wire, (*b*) by doubling the tension?
Describe any experiment which you would make in order to verify either of your statements.

2. How is it possible to cause a stretched string to emit a note having three times the frequency of its fundamental? Explain by the aid of a diagram the mode of vibration in this case.

3. According to what laws does the frequency of vibration of a stretched wire depend upon the length of the wire, the material of which it is composed, and the stretching force?
Describe experiments with the sonometer to verify these laws.

4. What do you understand by the harmonic modes of vibration of a stretched string?

When a violin string is bowed in the ordinary manner, several harmonic modes of vibration are induced in addition to the fundamental. How would you prove by experiment that this is the case?

5. How would you show that, when a stretched string is bowed, the octave above is generally sounded together with the fundamental note?

How should a stretched string be bowed to avoid sounding the octave?

6. Describe, giving possible numerical results, how you would prove that the frequency of vibration of a string vibrating transversely is proportional to the square root of the stretching force.

On increasing the weight stretching a given string by 2·5 kilogrammes, the frequency is altered in the ratio 3 : 2. Find the original stretching weight.

7. A string 50 cms. long, stretched by a weight of 10 kilogrammes, makes 256 transverse vibrations per second. How could the frequency of the note emitted be raised to 384 (1) by altering the length of the string, (2) by altering the stretching weight?

Could the string be made to emit a harmonic note of frequency 384 without altering either the length or the stretching weight? Explain fully.

8. How does the frequency of vibration of a stretched string depend upon its length, its mass, and the stretching force applied to it?

The upper end of a given copper wire is fixed to a peg, and it supports a weight at its lower end. When bowed it emits a certain note. Show that, if the wire were drawn out to four times its original length, it would, when supporting the same weight and bowed, emit a note an octave lower than the first.

9. How does the pitch of the note emitted by a wire vibrating transversely depend upon (1) its length and (2) the stretching weight? How would you test your statement by experiment?

How would you find by means of the sonometer whether two wires of different diameters were made of the same material?

10. A wire stretched on a sonometer is touched with a needle at a point whose distance is one-fourth of the length of the wire from one end and the shorter section is then lightly bowed. Describe and explain the state of vibration of the longer section, and show how to test your statement experimentally.

What happens if the needle is moved gradually to either side of the point?

CHAPTER XVI.

LONGITUDINAL VIBRATION OF RODS AND COLUMNS OF AIR IN PIPES.

106. The longitudinal vibration of a rod fixed at one end. The simplest mode of longitudinal vibration of a rod fixed at one end has already been described in detail in Art. **20**. It will be remembered that the rod in vibrating lengthens and shortens periodically so that all points in its length vibrate in the same period and in the same phase, each point vibrating about its normal position as centre. The amplitude of vibration is greatest at the free end, and diminishes along the rod towards the fixed end, so that if l denote the length of the rod, then the amplitude at any point at a distance x from the fixed end is given by $r \sin \frac{\pi x}{2l}$, where r is the maximum amplitude at the free end. It has also been explained that the strains produced in the rod during vibration are those of linear extension and compression. As the rod performs a complete vibration, first lengthening and then shortening, each section of the rod first undergoes a gradually increasing extension, then recovers its original state, then suffers a gradually increasing compression, and finally recovers its original state again. The range of strain, however, varies at different points; it is greatest at the fixed end and zero at the free end.

Generally then it may be said that the character of the vibration and the phase of vibration are the same at all points on the rod, but that the amplitude of vibration increases from zero at the fixed end to a maximum at the free end. Also the character of the strain cycle and the

phase of strain are the same at all points on the rod, but the range, or "amplitude," of the strain increases from zero at the free end to a maximum at the fixed end. In this mode of vibration, therefore, the fixed end of the rod is a *node*, or point of zero displacement and maximum range of strain, while the free end is an *antinode*, or point of maximum displacement and zero range of strain.

The period of vibration of the rod might be determined by the method of Art. **4**, by finding the force which acts on any transverse slice of the rod as the result of its displacement from its normal position. This method cannot, however, be given here.

If, however, we assume the velocity of propagation of a pulse of extension or compression along the rod as given by $V = \sqrt{M/D}$ (Art. **43**), the period of vibration of the rod can be determined in exactly the same way as that adopted in Art. **97** for the determination of the period of vibration of a string.

Imagine a pulse of compression to originate at the free end of the rod, say by tapping it at that end. This pulse travels down the rod to the fixed end and is there reflected with reversal of displacement, but, as explained in Art. **52**, with no reversal of the strain. It is therefore reflected as a pulse of compression and travels up to the free end, where it is reflected without reversal of displacement, but *with* reversal of strain. It therefore travels down the rod as a pulse of rarefaction to the fixed end, and is there reflected, still as a pulse of rarefaction, up to the free end, where, after reflection, it again starts down the rod as a pulse of compression.

If this pulse be supposed to travel up and down the rod by continued reflection from end to end it will be seen, as in the case of the string described in Art. **97**, that the motion produced in this way in the rod is of a periodic character, and that the period of the motion is the time taken by the pulse to travel, in either direction, from any point on the rod back through the same point in the same direction. That is, the period of the motion is the time taken by the pulse to travel over *four* times the length of the rod.

In general when any transverse section of the rod is displaced longitudinally the displacement originates a pulse of compression in the direction of displacement, and a pulse of rarefaction in the opposite direction. Each of these pulses travels up and down the rod and is reflected at the ends in the manner described above, and the two pulses meet, periodically, at their starting point, at intervals determined by the time either pulse takes to travel over four times the length of the rod. The motion of the rod is therefore in this case, as in the simpler case of a single travelling pulse, of a periodic character, the period of its motion being the time taken by a pulse of compression or rarefaction to travel over four times the length of the rod.

If now we suppose the initial pulse of compression to extend from the free end to the fixed end of the rod and imagine the sequence of displacements and stages of compression which constitute this pulse to travel up and down along the rod by repeated reflexion from the ends, it will be recognised that the periodic motion considered above becomes identical with the vibration of the rod as a whole when fixed at one end, as already described.

It will also be seen that the period of vibration of the rod will be the time taken by this pulse, that is by any particular displacement or stage of compression in it, to travel over four times the length of the rod, and that the frequency of the rod's vibration is therefore given by

$$n = V/4l = \frac{1}{4l}\sqrt{M/D}.$$

This result indicates that the frequency of vibration for a rod of *given material* is inversely proportional to its length, and for a rod of given length inversely proportional to the square root of its density.

The mode of longitudinal vibration considered above is the simplest mode of vibration possible for a rod fixed at one end and free at the other end. In order to determine the harmonic modes of vibration possible for a rod under these conditions, by dividing into segments, it must be remembered that the free end of the rod, being free, *must* be an *antinode*, and that the fixed end, being fixed, *must* be a *node*. That is, the length of the rod must always correspond to the distance from an antinode to a node.

Now the distance from an antinode to any node is evidently always an *odd number of quarter wave-lengths.* The harmonic modes of vibration possible for the rod are therefore those in which its length is occupied by one, three, five, seven, nine, or more quarter wave-lengths or half-segments as indicated in Fig. 79.

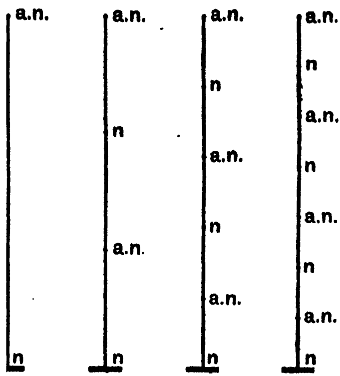


Fig. 79.

Hence, if a rod fixed at one end be suitably damped at a point at a distance x from the free end, and the portion of length x so separated be set in longitudinal vibration, it will vibrate in its simplest mode as a rod of length x fixed at one end and free at the other, with a frequency $V/4x$. Also, if x is an *odd* sub-multiple of l, that is if $l/x = p$ and p is integral and *odd*, then the whole rod is set in sympathetic vibration with the portion of length x in p *half-segments*, and the frequency for this mode of vibration is $pV/4l$. This result shows that for a rod of any given material fixed at one end the frequency of vibration is directly proportional to the number of half-segments in which it vibrates.

The frequencies of the modes of vibration possible for the rod are therefore, as indicated in Fig. 79, three, five, seven, nine, etc., times the frequency of the simplest or fundamental mode of vibration in one half-segment.

Again, if l denote the length of the rod and λ the wave length of longitudinal wave motion along the rod we evidently have

$l = 1 \cdot \frac{\lambda}{4}$, or $\lambda = 4\,l$ for the first or simplest mode of vibration

$= 3 \cdot \frac{\lambda}{4}$, or $\lambda = 4\,l/3$ for the second mode of vibration

$= 5 \cdot \frac{\lambda}{4}$, or $\lambda = 4\,l/5$ for the third mode of vibration

or, generally,

$l = (2n - 1)\frac{\lambda}{4}$, or $\lambda = 4l/(2n - 1)$ for the nth mode of vibration.

In determining the phase of vibration at any point in a rod vibrating in any one of these modes of vibration it must be remembered, as explained with reference to the vibration of a string, that, in stationary vibration, the phase of vibration is the same at all points in an internode *from node to node*, but that for points in adjacent internodes the phases of vibration are exactly opposed or differ by half a period.

When a rod fixed at one end vibrates in its simplest mode, in one half-segment, the note it emits is called the fundamental note of the rod. When it vibrates in three half-segments the frequency is three times that of the fundamental note, and the note it emits is a twelfth above the fundamental note. This note is evidently the second harmonic, the first harmonic, of twice the frequency of the fundamental note, being missing. Similarly when the rod vibrates in five half-segments the frequency is five times that for the fundamental note and the note is the fourth harmonic, two octaves and a third above the fundamental note. Generally, then, we may say that when the rod divides into n half-segments, *where n is odd*, the frequency is n times the frequency of the fundamental mode and the note given is the nth harmonic. That is, the modes of vibration possible to the rod can produce only the odd harmonics, of frequencies 1, 3, 5, 7 . . . $(2n - 1)$ times the frequency of the fundamental mode of vibration.

107. Longitudinal vibration of a rod fixed at its middle point and free at the ends. In the case of a rod clamped at the middle point and free at the ends each half will vibrate as a rod fixed at one end and free at the other. The middle point of the rod being a node, the phases of vibration in the two halves must always be exactly opposite, so that at any instant the particles in the respective halves are moving in opposite directions. Hence when the rod vibrates as a whole the two halves lengthen and shorten simultaneously, and the period of vibration of the rod is given by $n = \frac{V}{4l}$, where $2l$ denotes the length of the rod.

The modes of vibration of the rod under these conditions correspond exactly to those described above for a rod fixed at one end. Hence, if n denote the frequency of the fundamental note of the rod, the frequencies of the possible harmonics are given by $3n$, $5n$, $7n$, etc.

108. Longitudinal vibration of a rod fixed at both ends. If a rod is fixed at both ends and excited, as explained in Art. **20**, by rubbing along its middle portion, it can be set in longitudinal vibration. The simplest mode of vibration possible under these conditions is evidently that which gives an antinode at the middle of the rod and a node at each end. During vibration in this way the phase of vibration is evidently the same at all points on the rod; during one half-vibration the particles are all moving towards one end, and during the other half towards the other end. In each half of the rod, that is on opposite sides of the antinode, the phase of strain is opposite, so that when one half is in extension the other half is in compression. This mode of vibration of a rod is exactly analogous to the transverse vibration of a string stretched between two fixed points. The period of vibration of the rod is evidently given by $\frac{V}{2l}$, where $V = \sqrt{M/D}$, and, owing to the symmetry of its constraint, the other modes of vibration possible to the rod are obviously those in which the rod divides into two, three, four, or more equal

segments. The harmonics of the rod must therefore include both the even and odd harmonics. A stretched string can be set in longitudinal vibration in this way.

109. Compound modes of vibration of a rod. In general when a rod is excited to longitudinal vibration in any way it does not vibrate in any *one* of the modes described above, but in a complex mode compounded of some or all of the modes of which it is capable. In fact it is extremely difficult to so excite a rod as to set it in one definite mode of vibration. One particular mode may be made predominate, but the vibration is in general the resultant of all the modes of which the rod is capable under the conditions of its vibration. Hence when a rod is apparently sounding some particular harmonic this note is generally only the predominant component of a compound note including all the harmonics possible under the conditions of vibration. Hence, when a rod fixed at one end is apparently sounding its fundamental note, the note heard is really a compound of the fundamental note, in predominance, and the second, fourth, sixth, and higher harmonics. The harmonics present in this way with the fundamental note determine the quality of the compound note heard, and are sometimes called *overtones*. In the case of a rod fixed at one end the overtones are evidently only the odd harmonics. A note of this kind is generally of somewhat harsh quality for it lacks the strength and fulness given by the even harmonics which include the octave and double octave of the fundamental note.

The absence of the *even* harmonics from the overtones of a rod fixed at one end is evidently connected with the want of symmetry in the constraint of the rod; one end is fixed, the other is free, and as a result only odd harmonic modes of vibration are possible. If both ends were fixed or both free (without any other constraint) both odd and even harmonics would be found in the overtones of the rod.

Owing to the intensity of the stresses brought into action the frequency of a rod in longitudinal vibration is very high, so high that the pitch of the overtones soon passes beyond the limits of audibility. For example, the frequency of vibration for the fundamental note of a rod

of glass 20 cms. long fixed at one end is about 5,000 per second. The frequencies for the corresponding overtones are therefore 15,000, 25,000, 35,000, and so on. Of these probably only the first is within the audible limit for ordinary observers.

The longitudinal vibration of rods has practically no application in the construction of musical instruments.

110. Vibration of columns of air in pipes. In the consideration of the vibration of fluid columns it is usual to give most attention to air columns on account of the great importance of aerial vibrations in the construction of organ pipes and wind instruments generally. It should be remembered, however, that the vibration of an air column is a particular case of the vibration of a fluid column, and that whatever is said below of the vibration of air columns is true also of any fluid column, whether liquid or gaseous.

The longitudinal vibration of air columns is exactly similar to the longitudinal vibration of rods, but with one very important difference. In the case of a rod the strains accompanying the vibration are those of linear extension and linear compression, and the modulus of elasticity involved is therefore Young's modulus. In an air column, however, the strains accompanying vibration are, since air is a fluid possessing only volume elasticity, those of volume compression and rarefaction, and the modulus of elasticity involved is the modulus of volume elasticity under adiabatic conditions. As explained in Art. **12**, this modulus is measured by $1{\cdot}41\,P$, where P denotes the pressure of the air.

The strains in an air column in stationary vibration are therefore *volume* strains of compression and rarefaction. During compression at any point the pressure rises above its normal value and the density increases; during rarefaction at any point the pressure falls below its normal value and the density decreases. The range or "amplitude" of these changes of pressure and density depend upon the point at which they take place, being greatest at a node and zero at an antinode.

An air column also differs from a rod in the manner in which it can be constrained. It cannot be clamped at any point, but the end of a column can be fixed by closing the tube which contains it at that end. The end of a column at an open end of a tube or pipe is a free end because it is in free communication with the outer air, and cannot therefore be subject to variation of pressure. Any point in the column can also be made a "free" point, by putting it in communication with the outer air at that point, for example by making a hole in the tube.

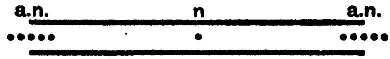


Fig. 80.

In any particular case the mode of vibration of an air column must evidently be determined by the nature of the constraints to which it is subject; a node must be formed at a closed end, an antinode at an open end and at any other "free" point in the tube. For example, the air in a tube or pipe open at both ends must vibrate with an antinode at each end, and *any* mode of vibration which satisfies this condition is a possible mode of vibration in this case. The simplest mode is evidently that which gives an antinode at each end and a node at the middle, as shown in Fig. 80.

If, however, a hole is made at the middle of the tube, a "free" point is established at that point, and the air must now vibrate with an antinode at each end and also at the middle point, and *any* mode of vibration which satisfies this condition is a possible mode of vibration in this case.

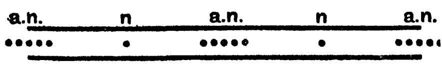


Fig. 81.

The simplest mode is evidently that which gives two nodes at points in the column distant one-quarter of its length from each end, as shown in Fig. 81.

The frequency of vibration in this case is double that of the first case, and the pitch of the note given by the

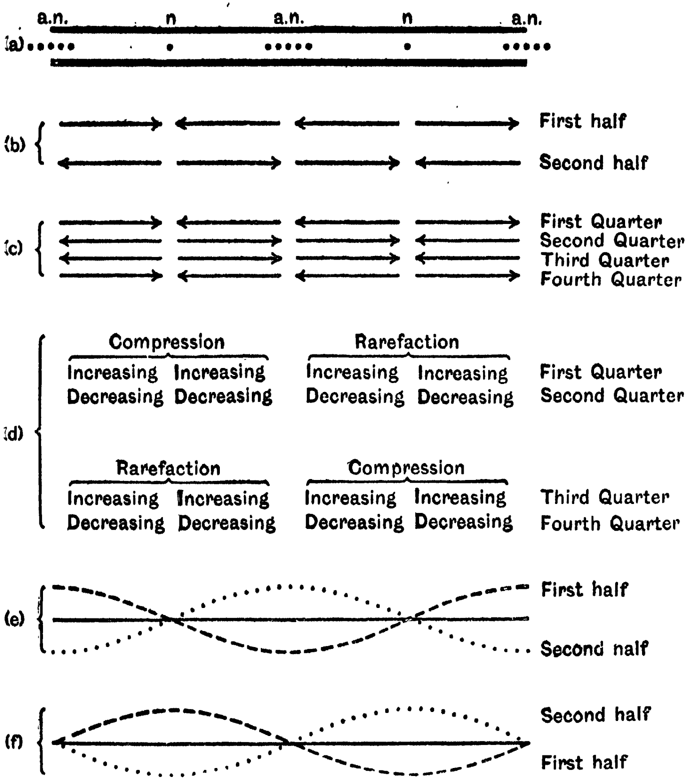
column is raised an octave by opening a hole at the middle point of the pipe. It can be understood from this how notes of different pitch can be obtained on the ordinary tin whistle or flute by opening and closing holes in the tube or barrel of the instrument. The note given by these instruments is produced by the longitudinal vibration of the air column in the barrel, and the pitch of the note is determined by the mode in which the column is made to vibrate. By opening and closing the holes in the barrel with the fingers, and by evoking where necessary some of the higher modes of vibration for a given constraint a considerable range of pitch can be adequately covered.

In order to specify the state of the air at any point in a column of air in longitudinal vibration, it is convenient to remember the following points, which have already been established.

In any internode, the half wave length from node to node, the phase of vibration of the particles is the same at all points, but the amplitude of vibration increases from zero at the nodes to a maximum at the antinode. Also in adjacent internodes, that is, on opposite sides of a node, the phases of vibration are exactly opposed.

In the half wave length from antinode to antinode the character of the strain (compression or rarefaction) is the same at all points, but the range or "amplitude" of the strain increases from zero at the antinodes to a maximum at the node. The range or "amplitude" of the strain at any point is determined by the extent of the change in density or the change in pressure which occurs at that point. Also in adjacent half wave lengths, measured from antinode to antinode, that is, on opposite sides of an antinode, the strains are opposite in character.

In any quarter wave length, measured from antinode to node, the strain is one of compression when the particles are displaced towards the node, and one of rarefaction when the particles are displaced away from the node. Also motion in the direction of displacement means an increasing strain, and motion in the direction



(*a*) Column of air in a pipe open at both ends in the mode of vibration corresponding to the production of its first harmonic.

(*b*) Arrows indicating the direction of displacement at all points along the column during a complete vibration.

(*c*) Arrows indicating the direction of motion at all points along the column during a complete vibration.

(*d*) Nature of the strain at all points along the column during a complete vibration.

(*e*) Curves showing the amplitude of displacement at different points along the column for a complete vibration. Displacements to the right are shown as positive ordinates.

(*f*) Curves showing the range or "amplitude" of the strain at different points along the column for a complete vibration. Compression strains are shown as positive ordinates.

Fig. 82.

action of this blast in the manner described below for the organ pipe. The vibration of the air column in a flute is another example of vibration set up by blowing across an opening in the pipe. The Pandean pipes also illustrate this method of exciting the vibration of an air column.

The most important application of the vibration of air columns in the construction of musical instruments is found in the organ pipe. An organ pipe usually takes the form of a cylindrical metal tube or a wooden tube of square section. One end of the tube is specially constructed, as described below, so that when air is blown through it from a bellows the air column in the tube is maintained in vibration. This end of the tube is in free communication with the air, and acts, with certain limitations, as an open end; the other end may be open as in *open pipes*, or closed by a tightly-fitting piston as in *stopped pipes*. The length of the pipe is determined by the wave length of the note to be produced, and the cross-section dimensions (which have no influence on the pitch of the note) are always small compared with the wave length.

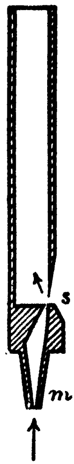


Fig. 85.

Fig. 85 shows a section of a wood organ pipe in which the construction of the lower end of the pipe is exhibited. The tapering mouthpiece *m* is made to rest in a corresponding socket, leading by a suitable channel to the wind chest fed by the bellows. It will be seen that the construction is such that the stream of air from the wind chest into the tube is forced through the narrow slit *s* as a thin ribbon-like sheet of air. This sheet is directed towards the thin razor edge which forms the upper boundary of the rectangular opening or *embouchure* which puts the air column in communication with the outer air and makes this end of the pipe an open end. The action of this thin ribbon-like blast of air from the slit *s* is usually supposed to initiate and maintain the vibration of the air column above it. The direction of the blast is such that a very slight inward deviation directs it into the pipe,

and a very slight outward deviation directs it into the outer air clear of the pipe. When directed into the pipe it must evidently produce compression, and when directed outwards it can, by suction, produce rarefaction at the lower end of the air column. If we imagine the blast to be directed into the pipe at any instant, it produces a compression which travels up the pipe to the other end, say a closed end, where it is reflected as a compression, and again returns to the point from which it started. Here it is reflected as a rarefaction, and at the same time pushes the air blast outwards, thereby increasing by the direct action of the blast the rarefaction set up by reflexion. This rarefaction now travels up to the outer end (closed), is reflected there as a rarefaction, and travels down again to the starting point, where it is reflected as a compression, and at the same time the air blast is forced into the tube by the excess of external pressure. The process already described is then repeated time after time. On this theory of the maintenance of the vibration of the air column in an organ pipe it is evident that the source of energy which maintains the continued vibration of the column is the kinetic energy of the air blast.

When the pipe is blown gently, so as just to sound steadily, it gives its fundamental note (with its characteristic overtones), but when blown strongly it may be made to give (as the predominant note) the second or even a higher harmonic. This is possibly explained by assuming that when the pipe is very strongly blown the action of the blast in producing compression or rarefaction is so vigorous that the sheet of air is driven out or into the tube before the compression or rarefaction last initiated by it has had time to travel up to the other end of the tube and back again. This means that the air column cannot vibrate in the fundamental mode, but that by vibrating in the second possible mode and so reducing the interval of time necessary between the inward and outward deflections of the blast, it may again bring the motion of the blast into step with its vibration and derive from it the energy necessary for the maintenance of the vibration.

It is also found that when an organ pipe sounding its fundamental note is blown more strongly, the pitch of the fundamental note rises appreciably before the first overtone is obtained. This is probably due to the action of the air blast being so increased that it is driven in or out of the tube a little sooner than it should be to coincide in its action with the return of the pulse reflected

from the top end. The result of this is that the lower limit of the air column in vibration is slightly raised, and the column is thereby shortened. This goes on until the fundamental mode of vibration breaks down and the second harmonic mode is established.

In tuning an organ pipe it is necessary to adjust in some way the length of the air column. In the case of stopped pipes this is readily done by means of the piston closing the upper end; the length of the air column can be increased or diminished and the pitch lowered or raised correspondingly by moving the piston up or down. In the case of open pipes there is generally a side opening covered by an adjustable plate near the open end of the pipe, and by adjusting the position of this plate the length of the vibrating column can be adjusted as may be required.

It is practically impossible to determine the length of the effective air column in a given organ pipe. The correction for the embouchure end is uncertain, and the correction for the other end of an open is also uncertain if it departs from the circular section, or if it carries any special tuning device.

The vibration of an air column may also be excited by *resonance*. For example, if a tuning fork in vibration is brought near the open end of a pipe for which the frequency of vibration of the air column is exactly the same as that of the fork, the column is at once set in vibration, and the note produced strongly reinforces the note of the fork with which it is in unison. This is the principle of action of the *resonance boxes* usually supplied with tuning-forks. The internal dimensions of the box are such that the frequency of vibration of the contained mass of air is the same as that of the fork mounted on it. The process by which the column of air in a closed tube, say, is set in resonant vibration by the tuning-fork may be followed briefly as follows.

Imagine the prong of the fork to vibrate from the extreme position at a (Fig. 86) down to the other extreme position at b, and back again to a. As the prong moves downwards from a to b, it starts a compression at the open end of the tube which travels down to the closed end, and after reflection there travels back up the tube still as a compression, and reaches the open end

again just as the prong reaches b. At this instant the reflection of the compression at the open end and the upward motion of the prong from b towards a *coincide* in starting a rarefaction at the open end. This rarefaction travels down to the closed end and after reflection there travels back up the tube still as a rarefaction, and reaches the open end again just as the prong reaches a.

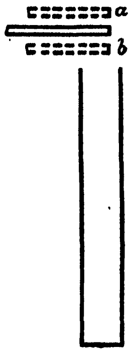


Fig. 86.

Here, again, the reflection of the rarefaction at the open end and the downward motion of the prong from a towards b *coincide* in starting a compression at the open end, and the process already described is repeated time after time. It is obvious that this coincidence between the direct and reflected pulses can take place only when the length of the pipe is such that the time taken by a pulse of compression or rarefaction in travelling over four times the length is the same as the period of the fork. That is, the frequency of the column of air in the pipe must, for the simplest case of resonance (Art. 24), be the same as that of the fork.

When the fork is removed the air column continues to vibrate only for the short time it takes to dissipate and radiate the small amount of vibration energy it possesses.

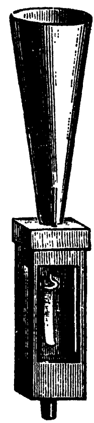


Fig. 87.

In some organ pipes the vibration of the air column is caused by resonance in response to the vibration of a *reed* fixed in the pipe and itself set in vibration by an air blast. The reed is a tongue of thin sheet metal fixed at one end with its length extending over a rectangular opening in the plate on which it is fixed. If the reed can pass freely through this opening it is called a *free reed*; if it overlaps it is called a *beating reed*. When air is blown through the opening the reed is set in transverse vibration, and, in the case of the reed organ pipe, this sets the air column of the pipe in vibration by resonance. Fig. 87 shows a reed organ pipe. The reed is usually

a free reed. Reed organ pipes are tuned by adjusting the length of the reed by means of a wire which presses on the reed and determines by its position the length free to vibrate.

114. Experimental methods of investigating the motion of the air in a pipe during longitudinal vibration. The motion of the air in an organ pipe during vibration is readily shown by a simple experiment due to Savart. A light ring, covered with tightly stretched paper or thin membrane, and carried like a scale pan by three light cords, is covered with a thin layer of fine dry sand and lowered into an open pipe in vibration. At an antinode, where there is maximum displacement of the air particles, the membrane moves up and down very rapidly, too rapidly for the motion to be observed, but the rattling of the sand on the membrane caused by the movement can be distinctly heard. At a node where there is practically no displacement the membrane is at rest and no rattling sound can be heard. As the membrane is moved from a node to an antinode the rattling at once begins, at first very faintly, but gradually attaining a maximum at the antinode. If the membrane is moved beyond this point to the next node the sound gradually dies away and ceases when the node is reached. If one side of the pipe is made of glass the agitation of the sand at an antinode can be distinctly seen. By means of this apparatus, therefore, the positions of the nodes and antinodes in a vibrating air column can be fixed, and the mode of vibration of the column determined.

Exp. 31. Suspend the membrane at the middle of an open pipe sounding its fundamental note. This point is a node and no rattling is heard. Blow the pipe more strongly until it sounds its first overtone. The middle point now becomes an antinode, and the sand is violently agitated and can be clearly heard rattling on the membrane.

The variations of pressure which accompany the strains of compression and rarefaction produced in the air column during vibration are most conveniently exhibited experimentally by means of *Koenig's manometric flames.* These

flames indicate variation of pressure by the following means. The gas-supply to the flame passes through the *manometric capsule* shown in section in Fig. 88. The gas enters by a tube, as at A, into a shallow cylindrical cavity C, about two or three centimetres in diameter, and passes from this cavity through the exit tube B to the burner, which is simply a fine pin-hole burner giving a small steady flame. One side of the cavity is closed by a thin membrane *m*, so that any variation of pressure at the outer surface of this membrane is at once indicated by the "jumping" of the manometric flame. Periodic variation

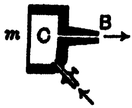


Fig. 88.

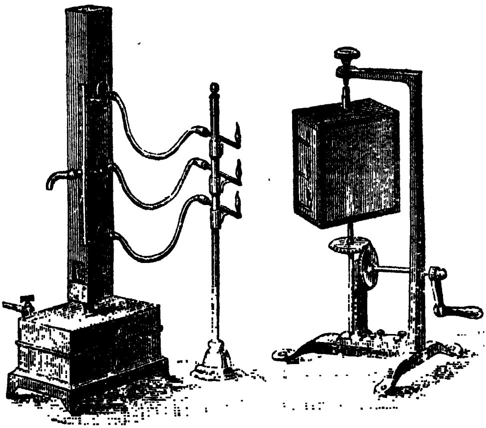


Fig. 89.

of the external pressure sets the membrane in vibration with the same period, and this, by its effect on the gas supply to the burner, causes the flame to jump up and down with corresponding frequency. The motion of the flame is much too rapid to be observed directly, although it is clearly indicated by the elongated and flickering appearance of the flame. If, however, the flame is examined

in a rotating mirror (Fig. 89), the successive images of the flame give a crooked band of light in the mirror, and the number of teeth seen in a given length of this band indicates the frequency of the flame's motion, and therefore of the periodic variation of pressure to which the motion is due.

In order to investigate the variation of pressure at a point in the air column in an organ pipe the manometric capsule is fitted to the wall of the pipe, either directly so that the membrane *m* forms a portion of the inner wall of the pipe, or by means of a receiving capsule fitted over the outer face of the membrane and communicating with the air in the pipe by means of a tube and a mouthpiece which can be inserted in a hole bored at any point in the pipe. Fig. 89 shows an organ pipe fitted directly with three manometric capsules fed by one supply tube and communicating with the three burners shown on the stand. The behaviour of the burners can then be simultaneously examined by means of the rotating mirror shown at the right of the figure.

When a manometric capsule is fixed at an antinode in a vibrating air column where there is no variation of pressure the flame of the attached burner is quite steady. When, however, the capsule is applied at a node where there is maximum change of pressure the flame jumps up and down with a frequency equal to that of the air column. At any point intermediate between a node and an antinode there is also periodic disturbance of the flame, but the amplitude of the disturbance is smaller than at the node, and the teeth of the image seen in the rotating mirror are smaller and less marked the further the point is from the node. The state of the air in a vibrating air column, as regards variation of pressure, can thus be studied at any point by means of Koenig's manometric flames, and the position of the node and antinodes for any particular mode of vibration easily determined.

Exp. 32. Fit an open organ pipe with three manometric capsules and burners, as shown in Fig. 89, at the points which divide its length into four equal parts, and make the pipe sound its fundamental note. It will be found that the middle flame indicates

a node at the middle point, but that the two other flames are practically undisturbed. Now make the pipe sound its first overtone; the top and bottom flames are disturbed, each indicating a node, and the middle flame is quite undisturbed, showing that it communicates with an antinode. These results confirm the descriptions already given of the modes of vibration of the column of air in an open pipe.

Manometric flames may also be used to compare the frequencies of two organ pipes. A capsule is applied at a node in each pipe, and the two flames are examined side by side in the rotating mirror. If it is found that n teeth in one image occupy the same length as n' teeth in the other image, the corresponding frequencies are evidently in the ratio n to n'.

The position of the nodes in a vibrating column can also be exhibited by means of fine dust, such as lycopodium powder or fine cork dust. If a thin line of dry lycopodium powder is arranged along a perfectly dry tube fixed horizontally, and the air column is set in vibration by resonance, the dust at the antinodes is violently displaced and collects in little characteristically shaped heaps at the nodes. The mode of vibration of the air column is thus clearly indicated and the wave length of the note can be measured. (See Art. **116.**)

The magnitude of the variation of pressure and the corresponding amplitude of displacement which takes place during the longitudinal vibration of an air column in an organ pipe have also been determined experimentally.

If a manometer or pressure gauge of the ordinary U-tube shape and containing water as the indicating liquid is applied at a node in the vibrating column by fitting the horizontal branch of the tube through a hole in the wall of the pipe, the manometer will be subject to a pressure which varies, with high frequency, above and below the atmospheric pressure. These variations will be so rapid that the water in the manometer cannot respond to them and will simply indicate the normal pressure about which the variations take place. If, however, the end of the tube fitted into the pipe is covered with a valve opening inwards only, then air from the pipe will pass into the manometer until the pressure in the latter is equal to the *maximum* pressure in the pipe. Similarly, if the valve is made to open outwards only, air passes from the manometer into the pipe until the pressure in the manometer is equal to the *minimum* pressure in the pipe.

By this means Kundt found that in a closed tube about a foot long the manometer indicated a variation of pressure at the node of about one-thirtieth of the atmospheric pressure. It can be calculated that this variation of pressure corresponds to a maximum amplitude of displacement at the antinode of about ·4 cm, that is, at the antinode the air particles, in this case, moved up and down through a total distance of about ·8 cm. or one-third of an inch.

115. Indirect methods of determining the velocity of sound. From the results given in this chapter it will be obvious that the velocity of sound in any material (in the limited sense explained below) may be determined if the frequency of longitudinal vibration of a rod of that material is known. Thus, if a rod of any material of length l, clamped at its middle point and set in longitudinal vibration, is found to have a frequency denoted by n, then, since $V = n\lambda$ (Art. **36**) and λ here equals $2l$, we have $V = 2nl$. This value of V, however, is strictly the velocity of longitudinal wave motion along the rod as given by $V = \sqrt{M/D}$, and although this is the only type of "sound" waves which can travel along the rod, it differs from the longitudinal wave motion in the material as a free medium which constitutes the true sound wave and which travels with the velocity given by $V = \sqrt{(E+\frac{4}{3}N)/D}$ as explained in Art. **43**. Further, if the material of the rod is of grained structure, like wood, the value obtained for V will evidently depend on how the rod is cut relatively to the grain.

It should be noticed in passing that if the value of V is taken as $2nl$ and if the value of D for the material be determined, the value of M can be calculated from the results of the experiment. That is, the experiment furnishes data for the determination of Young's modulus for the material.

Exp. 33. Take a deal rod, about 6 feet long, and clamp it firmly at its middle point in a vice. Excite the rod to longitudinal vibration by means of a leather rubber covered with powdered resin, and adjust its length (by cutting off, say, half an inch at a time from each end) until the pitch of the note it gives is the same as that of the note given by a tuning-fork of frequency 1024 per second. If the adjusted length of the rod is found to be 61 inches, then the

velocity of "sound" along this particular rod is given by $(2 \times 1024 \times 61)/12$ feet per second or about 10,400 feet per second.

Instead of adjusting the length of the rod in this way, it may be more convenient to compare the pitch of the note given by the rod with that of the note given by the fork by means of the sonometer wire, as explained in Art. **103**.

Although the results of this method in the case of a solid are of considerable indirect interest, they are of little value for the determination of the true velocity of sound in the solid. The value of the velocity of sound waves in an extended solid medium is, however, of theoretical interest only, and has not been studied experimentally.

In the case of air, however, the results of the method are of importance. The velocity of sound given by determining the frequency of vibration of an air column of known length in a pipe is the true velocity of sound in air along the pipe. The method can therefore be applied to determine the velocity of sound in air in pipes of different diameter, and by combining these results with the more extended results obtained by direct experiments, such as those of Regnault (Art. **85**), the velocity of sound in free air can be determined. This velocity is of practical interest and importance.

The application of the method to the longitudinal vibration of an air column is simplified by the fact that the air column, when properly adjusted in length, may be set in vibration by resonance to a standard tuning fork of known pitch. By this means the frequency of vibration of the air column can be determined with accuracy. In practice it is usual to work with a tube or pipe fixed vertically and closed at the bottom, so that the length of the air column in it can be adjusted by pouring water into the tube. In this way the length of the column is adjusted until it gives *maximum* resonance when the vibrating tuning-fork is held at the mouth of the tube. The point of maximum resonance can with care be found with considerable accuracy. Then when the adjustment is made the length of the pipe to the surface of the water and the diameter of the pipe are carefully measured. From the data so obtained the velocity of sound in air in the tube can be

calculated; for, if n denote the frequency of the fork, l the length of the pipe, measured as explained above, and r the radius of the tube, then, as already explained, $V = 4n(l + \cdot 6r)$.

Exp. 34. Take a narrow cylindrical jar or tube about 15 inches high, and hold a tuning-fork of pitch C (256) in vibration over the mouth of the jar. It will be found that the sound of the fork is faintly reinforced. If, however, water is poured carefully into the jar so as to shorten the air column, it will be found that the resonance to the fork at first increases and then decreases, and that there is a clearly defined point of maximum resonance. Adjust, therefore, for maximum resonance, several times until the ear is trained to the adjustment, and then take the mean value of the length of the jar from the mouth to the surface of the water for three or four careful adjustments. Measure the diameter of the jar. Assuming that the mean value found for l is 12·6 inches, and for r, 0·8 inch, then V is given by $V = 4 \times 256 \times 1{\cdot}09$ feet per second, or about 1116 feet per second. This is the velocity of sound in air of the temperature and humidity of the air in the jar along a tube 1·6 inches in diameter.

The experiment described above is the simplest form of experiments of this type, and in the form described is of considerable historical interest. More satisfactory results can, however, be obtained with the apparatus shown in Fig. 90.

The tube T and the reservoir R, mounted on a suitable stand, communicate, as shown, by a length of rubber tubing. The height of the water in the tube can evidently be raised or lowered by raising or lowering the reservoir.

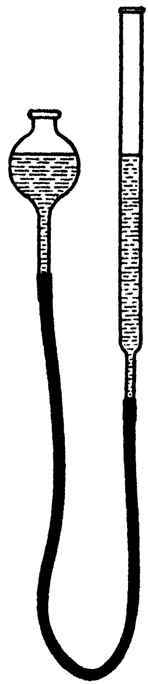


Fig. 90.

In the experiment described above the conditions are such that resonance is obgained when the tube sounds its fundamental note. In the general case, however, with a sufficiently long tube, resonance can be obtained for a series of regularly increasing lengths of the air column in the tube

The first, and shortest, length giving maximum resonance is that for which the fundamental note is in unison with the note of the exciting fork. The second length giving maximum resonance is that for which the *first overtone* is in unison with the note of the fork. In the case of a closed tube such as is used in this experiment, this second length will evidently be about three times the first length. Similarly, the third length giving maximum resonance will be that for which the *second overtone* is in unison with the note of the fork, and its length will be about five times the first length. In this way a series of lengths in the ratio $1 : 3 : 5 : 7 \ldots : (2n-1)$ may theoretically be found which respond either by the fundamental note, or by an overtone, to the note of the fork. In practice it is difficult to continue the series very far, and it is generally sufficient to obtain only the first two or three lengths. Kundt's experiment (Art. **116**), however, gives a case where the air column in the tube gives resonance by sounding a very high overtone.

For ordinary purposes an experiment with this apparatus may be carried out as follows. The level of the water in the tube is raised nearly to the top and then slowly lowered as the fork in vibration is held over the mouth of the tube. The first point of maximum resonance is thus determined, and the first length from the mouth of the tube to the level of the water measured. Let this length be denoted by l_1. The level of the water is now further lowered until the second point of maximum resonance is found and the second length of the air column is measured as before. Let this length be denoted by l_2. Now the distance $(l_2 - l_1)$ is evidently exactly half a wave length, for it is the distance from node to node when the air column of length l_2 is vibrating so as to give its first overtone. Hence the wave length corresponding to the note of the fork is given by $2(l_2 - l_1)$, and if n denote the frequency of the fork, the velocity of sound in air along the tube is given by $2n(l_2 - l_1)$.

It should be noticed that in this form of the experiment *the correction for the open end of the tube is eliminated.* This method has, in fact, been used for the experimental determination of the

correction necessary for the open end. Thus, since $(l_2 - l_1)$ denotes half a wave length, then $(l_2 - l_1)/2$ denotes the true length of a quarter wave length. But at the open end $(l_1 + x)$ denotes a quarter wave length if x is the proper correction for the open end. Hence we get

$$(l_2 - l_1)/2 = l_1 + x \text{ or } x = (l_2 - 3\,l_1)/2.$$

By this method Blaikley determined the correction at the open end to be given by $\cdot 576\,r$, where r denotes the radius of the tube.

Example. A tuning-fork of frequency 256 per second is held over the mouth of a resonance tube, 2·1 cms. in diameter, arranged as in Fig. 90. The first length for maximum resonance was found to be 32·6 cms., and the second length 99·0 cms.

From these data the half wave length is 66·4 cms., and the velocity of sound is given by $V = 2 \times 256 \times 66 \cdot 4$ cms. per second, or 34000 cms. per second.

The correction for the open end given by these data is 33·2 − 32·6 cms., or 0·6 cm.

The usual correction ($\cdot 6r$) gives ·63 cm.

116. Kundt's experiment. Kundt devised a very important method of comparing the velocity of sound in different gases and in rods of different materials with the velocity of sound in air as a standard. The method has also been adapted to apply to liquids and vapours.

In one of its simplest forms Kundt's apparatus for this method is shown in Fig. 91.

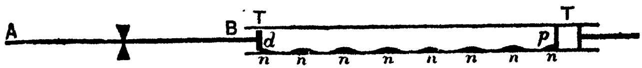


Fig. 91.

A rod, A B, of convenient length is clamped horizontally at its middle point. At the end B a light disc d, of cardboard or cork, is *securely and rigidly* attached to the rod by glue or sealing-wax. This disc is fitted into the mouth of the tube T T, which must be arranged so that the disc can move quite freely without touching the tube. The other end of the tube is closed by a movable cork piston p, which can be moved easily along the tube. For the purpose of an experiment the tube must be thoroughly dry, and a fine, even line of lycopodium powder or fine cork

dust arranged along its length. If, now, the rod A B is excited to longitudinal vibration by a suitable rubber, the air column in the tube can be set in vibration by resonance to the rod by moving the piston p in or out slowly until the length of the column is so adjusted that one of its overtones is of the same pitch as the note given by the rod. When the adjustment is such that the air column is set in strong resonant vibration, the line of fine powder in the tube will be violently disturbed every time the tube speaks, and when vibration ceases it is found arranged at the nodes in small heaps, showing characteristic transverse ridges, of which the central one may be taken to mark the position of the node.

In general, even with a fairly long rod, the pitch of the note is so high that the air column may have to divide into a large number of internodes in order to give an overtone of the same pitch. It is easy, therefore, to determine the average length of an internode with some accuracy, for the length occupied by from ten to twenty internodes is usually available for measurement.

The method adopted for comparing velocities with this apparatus is as follows.

The velocity of sound in the rod may be compared with the velocity of sound in air along the tube by comparing the wave length of the note given by the rod along the rod and along the air column. Now, if l denote the length of the rod, the wave length of the note *in the rod* is $2l$. Also if x is the average length of an internode of the air column when giving the same note as the rod, then the wave length of the note *in the air column* is $2x$. Hence, if n denote the frequency of the rod, the velocity of sound along the rod is given by $2nl$ and the velocity of sound in the air column is $2nx$; the velocity of sound in the rod is therefore $2nl/2nx$, or l/x times the velocity of sound in the air column.

The velocity of sound in any gas may be compared with the velocity of sound in air by adjusting for resonance, first with the tube T T full of air and then with it full of the gas considered, the same rod giving the same note being used in each case. Special precautions have to be

taken to prevent the escape of the gas. Then, if x denote the average length of an internode in air, and x' the average length of an internode in the gas, the ratio of the velocity of sound in the gas to the velocity in air is given by the ratios x'/x.

Example. In an experiment with Kundt's apparatus a glass rod, 80 cms. long, clamped at the middle, was used. The average length of the internodes in air was found to be 6·2 cms., and in carbonic acid gas 5·0 cms.

Here the wave length of the note is evidently 160 cms. in glass (along the rod), 12·4 cms. in air, and 10·0 cms. in carbonic acid gas. Hence the velocity of "sound" in the glass rod is 160/12·4, or 12·9 times the velocity of sound along the column of air in the resonance tube. Also, according to the data of the experiment, the velocity of sound along a column of carbonic acid gas in the tube is 10·0/12·4, or about 0·8 times the velocity in the air column.

The air column in the tube in these experiments vibrates as a column fixed at both ends. The end of the column adjacent to the disc on the end of the rod is not exactly a node, but a point near a node where the amplitude of vibration of the air is equal to the comparatively small amplitude of vibration of the disc. The real position of the node at this end is a short distance behind the disc.

It is of interest to note that this experiment is exactly analogous to Melde's experiment (Fig. 78), showing the transverse vibration of a string in resonant accord with the transverse vibration of a tuning-fork.

EXERCISES XVI.

1. Explain in what way the air vibrates in an open organ-pipe sounding its fundamental note. How would you show the state of motion of the air in the pipe?

2. What is the relation between the wave-length in air for a given note, and the length of the closed organ-pipe which resounds to it? Account for the difference in quality of notes of the same pitch from a closed and from an open organ-pipe.

3. In the case of a closed organ-pipe state clearly in what manner and direction the air particles move when the pipe sounds its fundamental note. How is the motion in the pipe produced?

4. What is the state of the air in an open pipe when the first harmonic is being produced in it?

5. If you blow across the open end of a key you can frequently obtain a shrill note. What connection is there between the length of the key and the shrillness of the note? By blowing very strongly, a note of much higher pitch may be obtained. Explain this.

6. When rods of the same length, but of different materials, are held in the middle and rubbed with a resined glove in the direction of their lengths, explain why musical notes are produced and why they are of different pitch.

7. Explain how to determine the velocity of sound in a solid, and state clearly on what properties the velocity depends.

8. Describe an organ flue-pipe. If two such pipes of the same length are sounded, the one open, the other closed, how do the notes differ from each other?

9. A brass tube closed at one end readily responds to a tuning-fork at the ordinary temperature, but not when heated considerably. Explain this. Could it be made to respond otherwise than by altering its length?

10. A small closed organ-pipe is connected by a long tube full of air with a bag of coal-gas. On forcing the air and gas gently through the pipe so as to elicit its fundamental note, how would you expect the pitch of the note to be affected when the coal-gas reached the pipe?

11. Taking the velocity of sound in air as 1120 ft. per second, find the length of the wave produced in air by a fork vibrating 384 times per second. Hence determine the length of an open organ-pipe which would yield the same note as the fork.

12. The length of a tube open at both ends that most loudly responds to a particular tuning-fork when the temperature is 15° C. is 6 ins. What is the frequency of the fork?

13. A tube closed at one end and filled with a gas gives the maximum resonance when 12 ins. long, with a fork of frequency 256. Calculate the speed of sound in the gas.

14. A wooden rod 5 ft. long held in the middle and rubbed with resined leather gives the same note as an open organ-pipe 4 ft. 3 ins. long. Find the speed of sound in the rod. Temp. 12° C.

15. A stopped organ-pipe 4 ft. long gives when filled with a certain gas the same note as an open pipe 5 ft. long filled with air. Calculate the speed of sound in the gas. Temp. 20° C.

EXAMINATION QUESTIONS.

1. A vibrating tuning-fork is held near the mouth of a cylindrical tube 32 cms. long closed at one end, and the tube is found to "speak." Explain clearly why this happens.

Assume the velocity of sound in air to be 340 metres per second, find the frequency of vibration of the fork.

2. A cylindrical tube 100 cms. long, closed at one end and of 1 cm. internal radius, is placed upright and filled with water, and a tuning-fork of frequency 510 is sounded continuously over its open end. Assuming the velocity of sound in the air to be 340 metres per second, describe exactly what you would expect to observe if the tube were gradually emptied.

3. An open organ-pipe emits a fundamental note of frequency 256 when sounded in air. Assuming the velocities of sound in air and coal-gas to be 350 and 500 metres per second respectively, find the pitch and wave-length of the note emitted by the organ-pipe when sounded in an atmosphere of coal-gas.

4. State how the frequency of the note emitted by an organ pipe depends upon (1) its length, (2) the pressure of the gas in which it is sounding, (3) the temperature of the gas, and (4) the nature of the gas.

Does the frequency depend upon the cross section of the pipe?

5. The disc of a siren possesses 32 holes and it is making 1050 revolutions per minute. Find the length of the open organ pipe which, sounding its fundamental, will emit the same note.

6. The end of one of the prongs of a tuning-fork is held over the mouth of a tube which can be raised or lowered in water. When the mouth of the tube is at a given height above the water the sound of the fork appears to swell out loudly. Carefully explain this.

Would the height be different if (*a*) the temperature of the air were higher, (*b*) if the air in the tube were replaced by carbonic acid gas? Give reasons for your answer.

7. State, and carefully account for, the difference in the mode of vibration of the air at the middle point of a closed and of an open organ pipe.

Describe a simple method of experimentally exhibiting this difference.

ANSWERS.

Exercises I. (*Page* 7.)

2. Acceleration $= \dfrac{v^2}{r} = \left(\dfrac{2\pi r}{\tau}\right)^2 \dfrac{1}{r} = \dfrac{4\pi^2 r}{\tau^2}$

$$= \frac{4 \times 10 \times 100}{1} = 4000 \frac{\text{cms.}}{\text{sec.}^2}.$$

Force in string = acceleration × mass of particle
= 40000 dynes.

3. In Fig. 4, if $Op = pB$, angle $a = 60°$. Therefore velocity of $p = v \sin 60° = v\sqrt{3}/2$.
Velocity at central point $O = v$.

4. Period $= 2\pi\sqrt{\dfrac{l}{g}}$, where l = length of string and g = acceleration due to gravity. 2 secs.

Exercises II. (*Page* 13.)

2. 1,010,000 dynes per sq. cm.

4. 500.

5. § 10 gives $M = \dfrac{WL}{l}$ when a = unity. If length is doubled, $l = L$; $\therefore M = W$.

Exercises III. (*Page* 20.)

2. $\frac{1}{200}$ sec.

3. Average velocity $= \dfrac{4 \times \text{amplitude}}{\text{period}}$; ∴ ratio of average velocities $= \frac{20}{10} \times \frac{20}{10} = 4$.

5. The curve is Fig. 9 shifted through a right angle.

6. $\frac{1}{4}$ or $\frac{3}{4}$.

7. (1) $\frac{2}{12}, \frac{4}{12}, \frac{8}{12}, \frac{10}{12}$; (2) $\frac{1}{12}, \frac{5}{12}, \frac{7}{12}, \frac{11}{12}$.

8. Using notation of Art. 3, we find that the maximum velocity is $r\omega$, also that the average velocity

$$= \frac{4 \times \text{amplitude}}{\text{period}} = \frac{4r}{2\pi/\omega} = \frac{2r\omega}{\pi};$$

$$\therefore \frac{\text{maximum velocity}}{\text{average velocity}} = \frac{r\omega}{2r\omega/\pi} = \frac{\pi}{2} = 1{\cdot}57.$$

9. Velocity of point at p (Fig. 5)

$$= r\omega \sin a = r \,.\, \frac{2\pi}{t} \sin a$$

$$= \frac{2\pi}{t} \,.\, r \sin a = \frac{2\pi}{t} r \cos (a + 90^\circ),$$

i.e. $\frac{2\pi}{t} \times$ displacement a quarter period later.

Acceleration of point at p (Fig. 5)

$$= r\omega^2 \cos a = \omega \,.\, r\omega \cos a = \frac{2\pi}{t} \,.\, r\omega \sin (a + 90^\circ),$$

i.e. $\frac{2\pi}{t} \times$ velocity a quarter of a period later.

11. (1) A straight line inclined at 45° to the N.S. or E.W. line; (2) a circle; (3) a straight line perpendicular to (1); (4) a circle described in the opposite direction to (2).

12. The path is an ellipse, the longer axis being parallel to the direction of motion of the point with the greater amplitude.

Exercises IV. (*Page* 33.)

2. For a given displacement the restoring force is greater with steel than with wood.

5. Amplitude of vibration $= r \sin \frac{\pi x}{2l}$. When $x = 0$, the amplitude $= 0$, which is true. When $x = l$, the amplitude $= r \sin \frac{\pi}{2} = r$, which is true.

6. See last paragraph, Art. 20.

10. In (1) the phases are equal, in (3) the phases are opposite. In (2) resonance occurs and the amplitude of vibration of the cork pendulum is very large.

11. The energy finally becomes heat.

12. Maximum velocity $= \dfrac{2\pi \times \text{amplitude}}{\text{period}} = 200\pi$ cms. per sec.

$$\text{Energy} = \tfrac{1}{2}\text{ mass} \times [\text{maximum velocity}]^2$$
$$= 400000\pi^2, \text{ or about 4 million ergs.}$$

Exercises V. (*Page* 43.)

4. Let XA (Fig. 23) represent the displacement curve of a point. Then the ordinate at $P = r \sin \omega t$, then the ordinate at $Q = r \sin \omega(t + t')$ where $t' = PQ$ and is small. Therefore

$$pq = r \sin \omega(t + t') - r \sin \omega t$$
$$= r\, 2 \cos \omega \left(t + \frac{t'}{2}\right) \sin \omega \frac{t'}{2}$$
$$= r\omega t' \cos \omega t \text{ very nearly, since } t' \text{ is small,}$$
$$= r\omega t' \sin (\omega t + 90^\circ).$$

The strain $= \dfrac{pq}{PQ} = \dfrac{r\omega t' \sin(\omega t + 90^\circ)}{t'} = r\omega \sin (\omega t + 90^\circ)$, therefore the strain diagram is a sine curve differing in phase by a quarter of a period from the displacement curve.

Exercises VI. (*Page* 57.)

4. 1000 cms. per second. 5. 135 cms. per second. 6. 110.

Exercises VII. (*Page* 62.)

4. 5095 metres per second.

5. $2{\cdot}06 \times 10^{10}$ dynes per sq. cm.

6. As in § 11 we have

$$PV^\gamma = (P + p)(V - v)^\gamma = (P + p)\, V^\gamma \left(1 - \frac{v}{V}\right)^\gamma,$$

$$\therefore\ P = (P + p)\left(1 - \gamma \frac{v}{V}\right) \text{ very nearly,}$$

$$\therefore\ P = P - P\gamma \frac{v}{V} + p \text{ very nearly,}$$

$$\text{or } P\gamma \frac{v}{V} = p,$$

$$\therefore \text{ the modulus of elasticity which } = \frac{pV}{v} = \gamma P.$$

8. $\dfrac{V_1}{V_2} = \sqrt{\dfrac{D_2}{D_1}}$, $\therefore$ velocity in oxygen = 320 metres per sec.

Exercises VIII. (*Page* 68.)

3. ·5 cm. $\dfrac{\text{Intensity at 100 metres}}{\text{Intensity at 200 metres}} = \dfrac{2^2}{1} = 4.$

6. The frequencies must be equal, or, as is proved in higher books on the subject, the frequency of the body must be a simple multiple of the frequency of the waves.

Exercises IX. (*Page* 77.)

1. and **2.** Sound waves are much longer than light waves.

7. 4.

Exercises XI. (*Page* 99.)

9. $\frac{3}{2}$ or a "fifth." **10.** 480. **11.** 10.

Exercises XII. (*Page* 117.)

4. 11200 feet. **13.** 5 secs.

14. 672 feet. **17.** 540.

Examination Questions. (*Page* 119.)

1. 5 cms., 30 cms. per second, $\frac{1}{6}$ sec.

Exercises XIII. (*Page* 129.)

5. 19·65° C. **6.** $\frac{20}{21}$ sec. **7.** 1144 ft. per sec.

8. 1130 ft./sec., 1120 ft./sec., 1150 ft./sec.

9. − 45° C., 15° C., 55° C., − 55° C.

10. 555 ft., 4440 ft., 2815 ft. **11.** 819° C.

12. 36·3. **13.** $4\frac{3}{8}$ ft., $4\frac{27}{64}$ ft. **14.** $4\frac{3}{8}$ ft.

15. $16\frac{17}{64}$ ft. **16.** 278. **17.** 900 ft./sec.

18. 10° C. **19.** −3° C. **20.** Four.

Exercises XIV. (*Page* 143.)

4. 1860. **5.** 256 ; 4·4 ft. **6.** 12·8.

10. 2. $\frac{9}{8} \times \frac{10}{9} \times \frac{16}{15} \times \frac{9}{8} \times \frac{10}{9} \times \frac{9}{8} \times \frac{16}{15} = 2.$

11. $\frac{5}{4}$, $\frac{4}{3}$, $\frac{16}{15}$. **12.** 384, 768, 192.

13. $213\frac{1}{3}$ (or $426\frac{2}{3}$ for the higher C).

EXAMINATION QUESTIONS. (*Page* 144.)

5. 1680 ft.
8. 275 vibrations per sec.; frequency (275) unchanged, velocity 4400 ft./sec., wave length 16 ft.
11. 560.

EXERCISES XV. (*Page* 169.)

2. The catgut.
3. Doubling its frequency.
4. Frequency trebled.
8. 156 lbs.
9. 53·4 inches.
10. $20\frac{1}{7}$ inches.
11. 240.
12. 80 lbs.

EXAMINATION QUESTIONS. (*Page* 170.)

1. (*a*) Doubled. (*b*) Increased to $\sqrt{2}$ times its original value.
5. Bow at the middle.
6. 2 kilos.
7. (1) Alter length to $33\frac{1}{3}$ cms. (2) Alter stretching weight to $22\frac{1}{4}$ kilos.
8. $n = \frac{1}{2l}\sqrt{\frac{t}{m}}$. l is increased fourfold and m is decreased to one quarter. $\therefore$ n is halved.

EXERCISES XVI. (*Page* 201.)

8. The open pipe gives a note an octave above the closed one.
11. 17·5 ins.
12. 1120.
13. 1024 ft./sec.
14. 1310 ft./sec.
15. 1808 ft./sec.

EXAMINATION QUESTIONS. (*Page* 203.)

1. 266.
2. Maximum resonance when air column is 16 cms., $49\frac{1}{3}$ cms., and $82\frac{2}{3}$ cms. long.
3. Frequency, 366 (nearly); wave length, 136·7 cms. (Same as in air.)
4. $n \propto \frac{\sqrt{\text{absolute temperature}}}{\text{length}}$. A wider pipe is effectively a little longer than a narrow one of the same length because it has a greater end correction.
5. 1 foot.
6. (*a*) Yes, greater. (*b*) Yes, less.
7. In the middle of a closed tube the air is always at rest, in the middle of an open tube the air is always in motion except at two instants during one complete vibration.

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